The effect of errors in the published heights of National GPS Network Active stations

1 Introduction

In 2004 errors were discovered in the published height coordinates of the Active stations in the Ordnance Survey National GPS Network. These errors have been corrected and new coordinates for the Active stations were issued in September 2004. An explanation of the cause of the errors and their likely effect on the computed heights of user's stations is given here.

Ordnance Survey is grateful to the Institute of Engineering Surveying & Space Geodesy (IESSG) at the University of Nottingham for their assistance in isolating the errors and testing their magnitude.

2 The cause of the error

A GPS baseline is measured between the phase centres of the two antennas. To relate this baseline to the station markers the vertical height from the station marker to the antenna phase centre must be known. The phase centre is not a physical point that can be measured to, so the offset of the phase centre from a physical point on the antenna must be accounted for during processing.

The antenna heights of all Ordnance Survey Active stations are measured vertically from the station marker to the Antenna Reference Point (ARP) for each antenna, which is usually the base of the antenna mount. For all the Active stations in the Ordnance Survey National GPS Network the station marker is the ARP, so the antenna height is 0.000 m.

The offset from the ARP to the phase centre is then added to the antenna height in the processing software to give the height of the phase centre above the station marker.

The main phase centre offset component is vertical (up) but there are also small horizontal offsets to north and east. There are actually two phase centres in a dual-frequency antenna – one for the L1 frequency and the other for L2, and each phase centre has a different offset.

The phase centre applicable at the time of observation of the satellite signal also varies depending on the elevation of the satellite. Hence, phase centre corrections must be applied in the processing to take this into account. These corrections usually take the form of a mean offset value and a look-up table with additional elevation dependent values quoted for every 5 degrees of elevation, with separate tables for L1 and L2 frequencies.

During the processing to determine the published coordinates of the Active stations it was an error in the input of the elevation dependent values that caused the height errors.

The values come from a file supplied by the International GPS Service (IGS). The file is *ftp://igscb.jpl.nasa.gov/pub/station/general/igs_01.pcv*. The elevation dependent corrections are quoted every 5 degrees starting at 90 degrees elevation (i.e. vertically upwards) down to 0 degrees elevation (i.e. horizontal).

The error in inputting these values occurred when they were transferred to a different format for use in the GPS processing software used by Ordnance Survey. When the values were transferred to the new format their order was mistakenly reversed so, for example the correction relating to 90 degrees elevation was being applied in the software at 0 degrees.

3 The magnitude of the errors

The error in the elevation dependent corrections translated almost directly to an error in station height. Very small errors were introduced to the horizontal coordinate components but these were of the order of 2 mm or less and well inside the expected noise level of the Active network coordinates.

The height error is different at each station but is correlated with antenna type. The change in the height at each station is given in the table below. The mean height error for each antenna type is also given.

Station	Antenna Type		Station Height Error (m)	Mean Height Error for Antenna Type (m)
DROI	ASH700936E	SNOW	0.012	
INVE	ASH700936E	SNOW	0.013	
KING	ASH700936E	SNOW	0.010	
LEED	ASH700936E	SNOW	0.012	
LOND	ASH700936E	SNOW	0.012	0.012
NEWC	ASH700936E	SNOW	0.012	
NORT	ASH700936E	SNOW	0.010	
NOTT	ASH700936E	SNOW	0.011	
OSHQ	ASH700936E	SNOW	0.013	
BLAC	LEIAT504	LEIS	0.005	
BLAK	LEIAT504	LEIS	0.006	
CARL	LEIAT504	LEIS	0.005	
CARM	LEIAT504	LEIS	0.006	
COLC	LEIAT504	LEIS	0.005	
DARE	LEIAT504	LEIS	0.006	
EDIN	LEIAT504	LEIS	0.006	
GLAS	LEIAT504	LEIS	0.006	0.006
IOMN	LEIAT504	LEIS	0.005	
IOMS	LEIAT504	LEIS	0.006	
MALG	LEIAT504	LEIS	0.007	
PLYM	LEIAT504	LEIS	0.006	
SCAR	LEIAT504	LEIS	0.010	
TAUN	LEIAT504	LEIS	0.011	
THUR	LEIAT504	LEIS	0.008	
BUT1	TRM33429.00+GP	DOME	-0.021	-0.023
FLA1	TRM33429.00+GP	DOME	-0.026	
GIR1	TRM33429.00+GP	DOME	-0.023	
LIZ1	TRM33429.00+GP	DOME	-0.018	
LYN1	TRM33429.00+GP	DOME	-0.024	
NAS1	TRM33429.00+GP	DOME	-0.025	
NFO1	TRM33429.00+GP	DOME	-0.029	
SCP1	TRM33429.00+GP	DOME	-0.015	

STI1	TRM33429.00+GP DOME	-0.025	
SUM1	TRM33429.00+GP DOME	-0.017	
WOR1	TRM33429.00+GP DOME	-0.021	

The “sense” of the above error values is the old incorrect height *minus* the new correct height (assumed to be the "truth"), i.e. the given error represents the change that needs to be applied to the new height of a station in order to obtain the old height. Therefore the old published heights for the stations with Ashtech and Leica antennas were too high by 12 mm and 6 mm respectively. The old published heights for the stations with Trimble antennas (the General Lighthouse Authority stations) were too low by 23 mm.

4 The effect of the errors on user’s coordinates

It is impossible to calculate the exact effect of these errors on the coordinates of a user’s station without readjusting the user’s computed GPS baselines. However by making various assumptions a guide to the expected error for a particular combination of antenna types can be given.

Each processed GPS baseline, computed from a user’s station to an Active station, can be used to calculate an ellipsoidal height difference between the two stations. The height of the Active station (assuming it is held fixed in the user’s adjustment) plus the height difference gives a value for the height of the user’s station.

$$H_U = H_A + \Delta H_{AU}$$

where: H_U = Height of user’s station;
H_A = Height of Active station;
ΔH_{AU} = Height difference from Active station to user’s station

When several Active stations are used the final value for H_U can be expressed as:

$$H_U = \frac{\sum_1^i \left[H_{Ai} + \Delta H_{AiU}\right]}{i}$$

where: H_{Ai} = Height of an Active station;
ΔH_{AiU} = Height difference from an Active station to user’s station;
i = Number of active stations used.

In reality the values of ΔH_{AiU} would be weighted according to the quality of the computed baselines. However, in trying to compute a guide to the error at a user’s station, the quality of an actual baseline cannot be known, so equal weighting of the values for ΔH_{AiU} has to be assumed.

The error in the Active station heights can now be introduced to the equation:

$$H_U = \frac{\sum_1^i \left[\left(H_{Ai} + e_i\right) + \Delta H_{AiU}\right]}{i} = \frac{\sum_1^i \left[H_{Ai} + \Delta H_{AiU}\right]}{i} + \frac{\sum_1^i \left[e_i\right]}{i}$$

where: e_i = the error in the Active station height.

So for a particular combination of Active station antennas A_1 to A_i an estimate of the total error introduced at the user's station = $\frac{\sum_1^i [e_i]}{i}$. Again the sense of this error is *from* the correct height *to* the incorrect height.

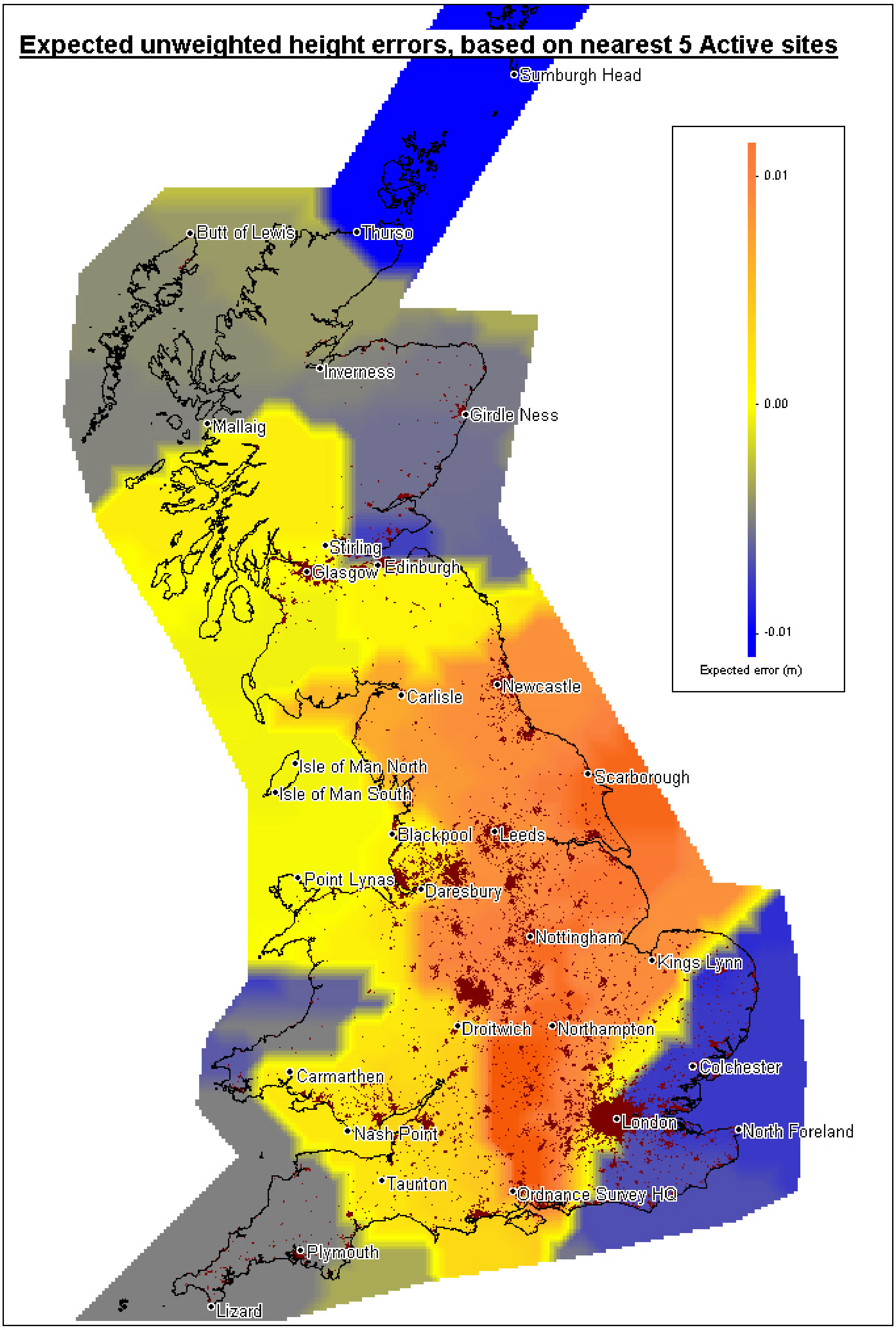
Using the above formula and the mean error values per antenna type and assuming the nearest Actives to a site are used, estimates of the total error for different combinations of antennas have been calculated and are shown in the table below.

Number of Sites with a particular antenna			Total No. sites used	Expected error	Is antenna combination expected in GB?
Ashtech	**Leica**	**Trimble**			
5	0	0	5	0.0115	Yes
4	1	0	5	0.0105	Yes
3	2	0	5	0.0094	Yes
2	3	0	5	0.0083	Yes
1	4	0	5	0.0072	Yes
0	5	0	5	0.0061	Yes
4	0	1	5	0.0047	Yes
3	1	1	5	0.0036	Yes
2	2	1	5	0.0025	Yes
1	3	1	5	0.0015	Yes
0	4	1	5	0.0004	Yes
3	0	2	5	-0.0021	No
2	1	2	5	-0.0032	Yes
1	2	2	5	-0.0043	Yes
0	3	2	5	-0.0054	Yes
2	0	3	5	-0.0090	No
1	1	3	5	-0.0101	Yes
0	2	3	5	-0.0111	Yes
1	0	4	5	-0.0158	No
0	1	4	5	-0.0169	No
0	0	5	5	-0.0226	No
4	0	0	4	0.0115	Yes
3	1	0	4	0.0102	Yes
2	2	0	4	0.0088	Yes
1	3	0	4	0.0075	Yes
0	4	0	4	0.0061	Yes
3	0	1	4	0.0030	Yes
2	1	1	4	0.0016	Yes
1	2	1	4	0.0003	Yes
0	3	1	4	-0.0011	Yes
2	0	2	4	-0.0055	Yes
1	1	2	4	-0.0069	Yes
0	2	2	4	-0.0083	Yes
1	0	3	4	-0.0141	No
0	1	3	4	-0.0154	Yes
0	0	4	4	-0.0226	No
3	0	0	3	0.0115	Yes

Number of Sites with a particular antenna			Total No. sites used	Expected error	Is antenna combination expected in GB?
Ashtech	Leica	Trimble			
2	1	0	3	0.0097	Yes
1	2	0	3	0.0079	Yes
0	3	0	3	0.0061	Yes
2	0	1	3	0.0001	Yes
1	1	1	3	-0.0017	Yes
0	2	1	3	-0.0035	Yes
1	0	2	3	-0.0112	Yes
0	1	2	3	-0.0131	Yes
0	0	3	3	-0.0226	No
2	0	0	2	0.0115	Yes
1	1	0	2	0.0088	Yes
0	2	0	2	0.0061	Yes
1	0	1	2	-0.0055	Yes
0	1	1	2	-0.0083	Yes
0	0	2	2	-0.0226	Yes
1	0	0	1	0.0115	Yes
0	1	0	1	0.0061	Yes
0	0	1	1	-0.0226	Yes

The “Is antenna combination expected in GB” column is based on computing the nearest antennas at every point in a 20 km grid covering the whole country from OSGB36 National Grid coordinates 0,0 to 700000,1260000.

The same algorithm that was used to produce the figures in the table above was used to calculate the expected error when using the nearest 5 Active stations at every point in the 20 km grid. In this case the combined errors were calculated using the individual Active station errors not the mean error per antenna type. These errors were then represented by colours and plotted on a map of GB to indicate where a particular error might be expected. This map is shown below and the errors range from -11 mm to +11 mm. It can be seen that when using the nearest 5 Active stations, the expected error is less than 10 mm in many locations.


Expected unweighted height errors, based on nearest 5 Active sites
Sumburgh Head
Butt of Lewis
Thurso
Inverness
Girdle Ness
Mallaig
Stirling
Glasgow
Edinburgh
Carlisle
Newcastle
Isle of Man North
Isle of Man South
Scarborough
Blackpool
Leeds
Point Lynas
Daresbury
Nottingham
Kings Lynn
Droitwich
Northampton
Colchester
Carmarthen
London
North Foreland
Nash Point
Taunton
Ordnance Survey HQ
Plymouth
Lizard
0.01
0.00
-0.01
Expected error (m)