The Irish Grid

A Description of the Co-ordinate Reference System

Published by
Director, Ordnance Survey of Ireland, Dublin
and
Chief Executive
Ordnance Survey of Northern Ireland, Belfast

CONTENTS

List of Diagrams

1.0 INTRODUCTION

This joint technical information paper describes the basis and derivation of the co-ordinate reference system used in Ireland. This system, generally known as the Irish Grid, is shared by the Ordnance Survey of Ireland (OSi) and Ordnance Survey of Northern Ireland (OSNI), based in Dublin and Belfast respectively, and is widely used throughout Ireland to describe positions on the earth's surface in an unique and unambiguous manner.

The availability of digital mapping and the potential for satellite positioning systems have increased many users interest in the Irish Grid and how it relates to other positioning systems - in particular the Global Positioning System (GPS). This is the first in a series of joint technical papers aimed at informing OS data users and the public in general alike on a number of technical matters. The previous publication 'Ordnance Survey Tables for the Transverse Mercator Projection of Ireland' (1971), is now withdrawn, and is replaced by the present paper.

Basic geodetic concepts are introduced, and an outline of the development of the Irish Grid given. Standard formulae and constants are provided in a technical data section, followed by worked examples of computing in the co-ordinate system.

Acknowledgements

Some of the formulae and examples are derived from the booklet 'The Ellipsoid and the Transverse Mercator Projection', published by OSGB as Geodetic Information Paper No 1, © Ordnance Survey (Great Britain), Southampton, 1995, reference **[1]**. The authors are grateful to Mr. F. Prendergast, Dublin Institute of Technology, for checking the example computations.

2.0 CONCEPTS

2.1 Introduction

This section provides an introduction to the geodetic concepts behind the Irish Grid co-ordinate reference system. For a more detailed treatment, particularly concerning geodesy and the mathematics involved, the reader is directed to, reference **[2]**, **[3] and [4]**.

2.2 The Shape of the Earth

The earth is approximately a sphere, but with a varied and irregular surface. One of the basic aims of the science of geodesy is to determine the position of points on (as well as above or below) the earth's surface, in a way which is unique and unambiguous. Although no simple mathematical model exists to cope with the variations in the earth's true shape, historical attempts to define its size and shape discovered that it approximates to an oblate spheroid eg. a sphere slightly squashed at the poles, or an ellipse rotated about the semi-minor axis which is aligned to the axis of rotation of the earth, an *ellipsoid*. In fact, points on the earth's surface can deviate vertically by up to 9 km from the best fitting global *ellipsoid.*

2.3 The Geoid

So what is the true shape of the earth? The *geoid* is defined as that equipotential surface of the earths gravitation and rotation, which on average coincides with mean sea level in the open ocean, i.e. a gravimetric equipotential surface approximating to mean sea level. This may be visualised conceptually as a surface which coincides with mean sea level (imagining there was no land), where the effects of non-gravitational forces, such as tides, currents and meteorological effects, are removed. However, there are local gravitational anomalies to this simplistic concept, due to land mass, noticed particularly in mountainous areas, and these can distort the shape of the geoid locally.

The *geoid* is of fundamental importance in determining positions on the earth's surface as most measurements are made with reference to this surface. For instance, heights are referred to mean sea level (which is effectively the *geoid*), and many measurement devices, such as geodetic theodolites and levels, use the force of gravity to enable them to determine directions. Furthermore, satellite systems operate within an environment directly influenced by gravity. The *geoid* is not a simple mathematical surface (although it can be mathematically modelled), but can deviate vertically by up to 100m from an *ellipsoid*, largely due to variations in gravity around the globe.

2.4 The Reference Ellipsoid

Ellipsoids are generally a good approximation to the shape of the *geoid*. They are simple to define mathematically and have been used in classical geodesy for over 200 years to provide a figure of the earth on which positions may be given in terms of latitude, longitude and height above the *ellipsoidal* surface. An *ellipsoid* thus used, is termed a *reference ellipsoid*. As stated before, the shape of the *geoid* varies around the globe, therefore, different sized *ellipsoids* have been used for different regions. Each is chosen to achieve the 'best fit' to the *geoid* as closely as measurement technologies and computational abilities allowed at the time they were established. For example, an *ellipsoid* which provides a good fit of the *geoid* over the whole globe is not necessarily the most suitable for North America, and neither would be the most appropriate for Ireland, see **Diagram 1** for an exaggerated depiction.

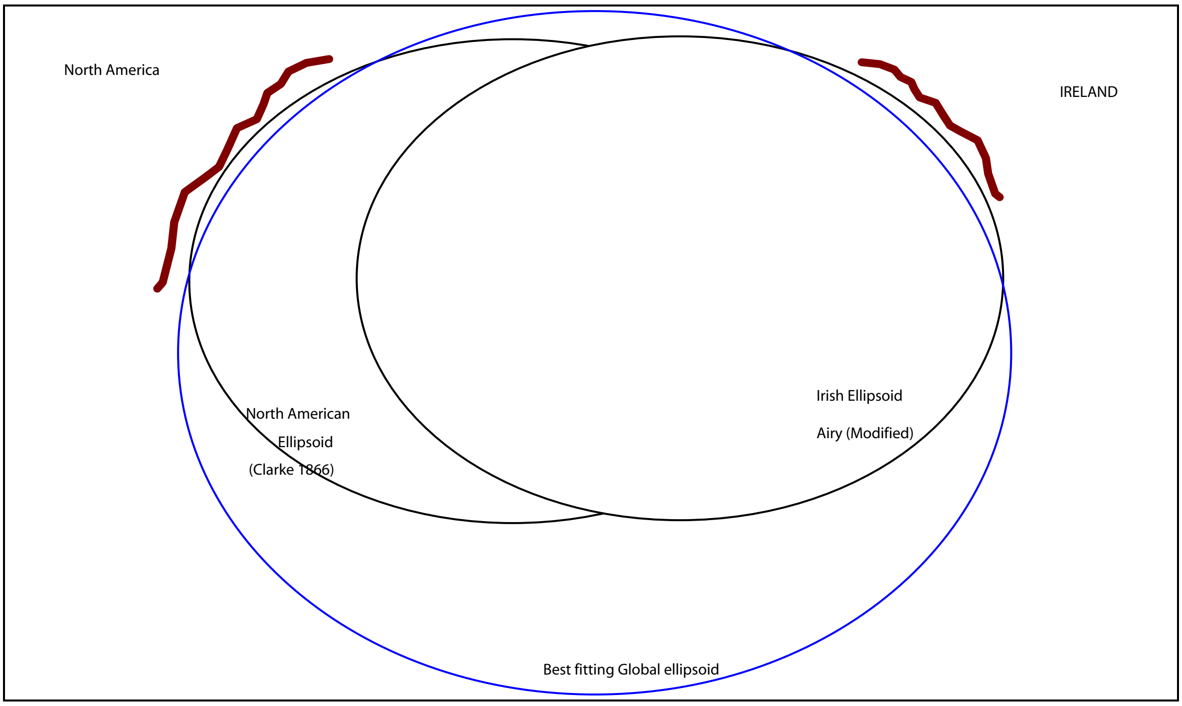


Diagram 1 ***: Exaggerated diagram of regional ellipsoids.***

This shows an imaginary section through the earth, with the regional ellipsoids positioned to closely fit the geoids of the country concerned. In a similar way, the best fitting global ellipsoid would not necessarily be the most appropriate for either region.

2.5 Geodetic Datum

Thus, there are many different ellipsoids on which positions may be expressed. The size, shape and positioning of the *ellipsoidal* reference system with respect to the area of interest is largely arbitrary, and determined in different ways around the globe. The defining parameters of such a reference system are known as the *geodetic datum*. The *geodetic datum* may be defined by the following constants:

- the size and shape of the *ellipsoid*, usually expressed as the semi-major axis (**a**), and the flattening (**f**) or eccentricity squared ($\mathbf{e^2}$), see **Section 5.0**. There are a number of techniques used to determine the 'best fit' *ellipsoid* for an area. Historically, in Great Britain and Ireland a geodetic triangulation network was used;

- the direction of the minor axis of the *ellipsoid*. This is classically defined as being parallel to the mean spin axis of the earth, and achieved by comparing the observed astronomic bearing of a line (say in a geodetic triangulation network) with its calculated ellipsoidal bearing, satisfying the *Laplace*[1] condition, and adjusting the triangulation network as appropriate;

- the position of its centre, either implied by adopting a geodetic latitude and longitude (ϕ, λ) and *geoid-ellipsoid separation* (**N**) (see **Section 2.7** below) at one, or more triangulation points (datum stations), or in absolute terms with reference to the centre of mass of the earth;

- the zero of longitude (conventionally the Greenwich Meridian).

The manner in which the *geodetic datum* is defined varies from country to country (or region to region), usually through survey observation's, adoption of international standards, or acceptance of some form of historical convention.

2.6 Height Datum

The *geoid* is an irregular shape and because of this, the *geoid's* surface is not generally parallel to the *ellipsoidal* surface. It is therefore usual to fix the *geoid* at one location, usually some form of reference mark at which height above mean sea level has been determined, and refer heights to this point for practical purposes - this is known as the *height* or *vertical datum*.

Although the relationship between the *geoid* and *ellipsoid* is known at this point, and may be known at certain other points, the separation is not constant and furthermore can vary considerably, depending upon the nature of the *geoid* in the area of interest. A mathematical model of the variation may be required in order to determine the separation elsewhere. By choosing the best fitting *ellipsoid* this separation can, in certain circumstances, be ignored. However, with global geocentric *ellipsoids*, or in areas of significant terrain variation, the separation and variation can be significant, particularly when transforming positions between reference systems. In these circumstances a good mathematical model of the *geoid* is important.

2.7 Ellipsoidal Reference System

The position of a point **(P)** may be measured on the earth's surface and given a geographical co-ordinate ie. latitude (ϕ) and a longitude (λ), in terms of a particular *ellipsoidal reference system*, see **Diagram 2**. By projecting down onto the *ellipsoid's* surface along a line perpendicular to the ellipsoid surface, the *normal* is described, see **Diagram 3**. It should be noted that the direction of the true vertical ie. the direction of gravity, may be slightly different from the *normal*, and this difference in direction is termed the *deflection of the vertical*, which for most practical purposes may be ignored. The distance along the *normal* to the ellipsoid is termed the *ellipsoidal height*, usually in geodesy designated by the letter ***h***, and fixes the point (**P**) in 3 dimensional space. Height above sea level, in geodesy termed the *orthometric height*, and designated by ***H***, is the most useful height for practical purposes, and is usually measured by geodetic levelling networks. The separation between the *ellipsoid* and *geoid* along the *normal* is generally

[1] Laplace points provide an independent determination of ellipsoidal bearings, which may be then compared to the computed bearing to determine any errors in the angles used to carry bearings forward through a survey network. Classically it is usual in geodetic networks to include such points in the final adjustment.

known as the *geoid-ellipsoid separation*, in geodesy designated the convention ***N***, and should be recognised as an important element in the computational process. Thus, positions are given in terms of latitude, longitude and *ellipsoidal heights*.

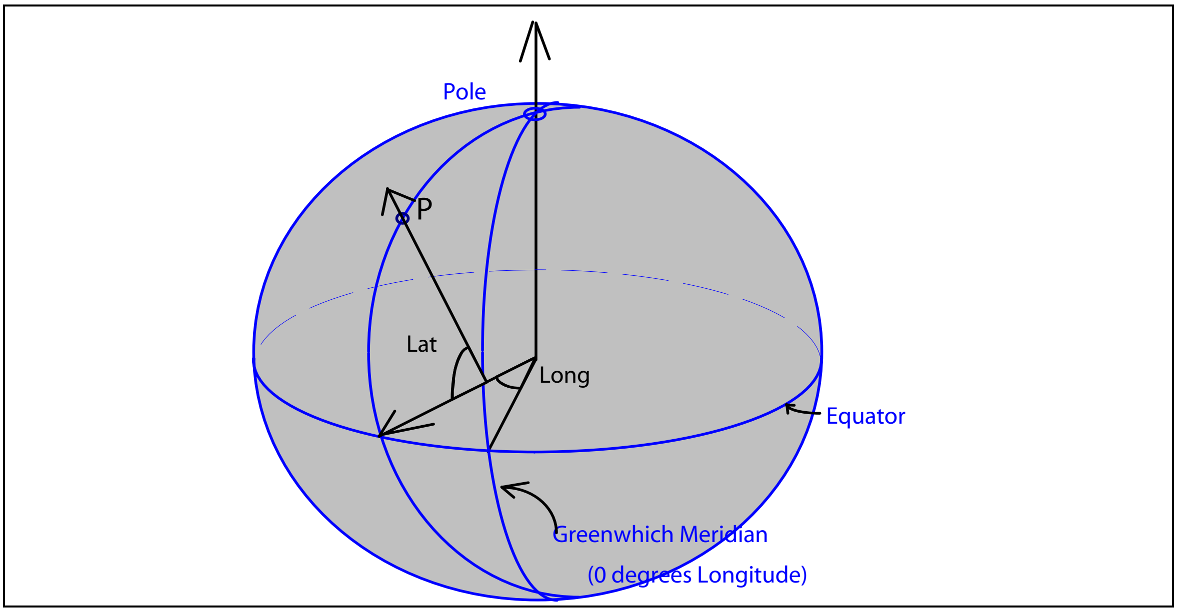


Diagram 2 : *Ellipsoidal Reference system.*

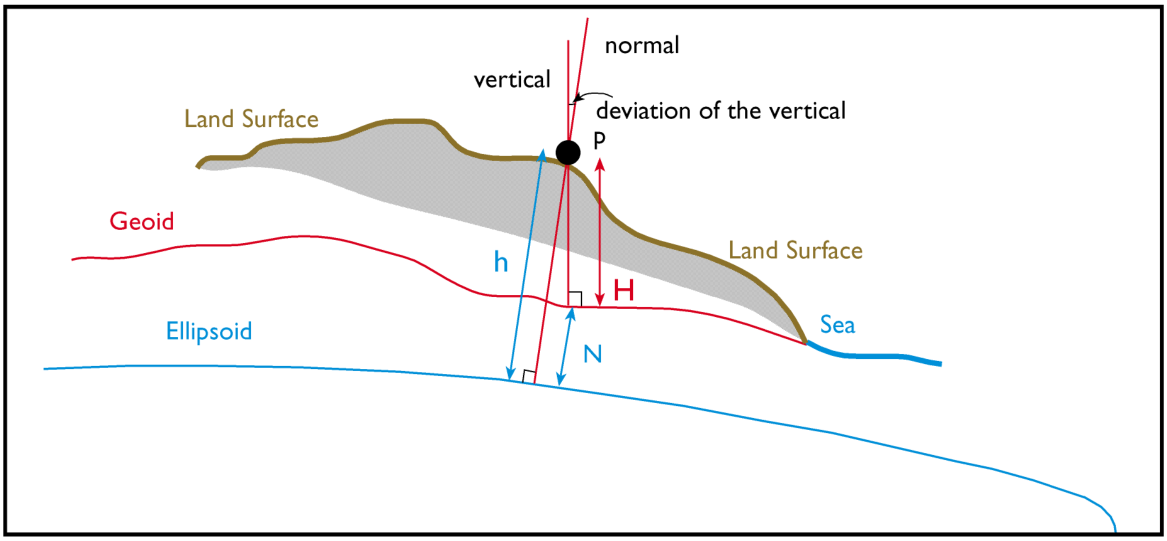


Diagram 3: *The relationship between the ellipsoid, the geoid, land surface, normal and true vertical*

2.8 Cartesian Reference System

Positions may be given in absolute terms, relative to the earth's centre of mass, or an assumed centre, as implied by a *geodetic datum*. In a Cartesian Reference System positions are defined in 3 dimensional space by an X, Y, Z co-ordinate triplet, normally described in units of International Metres, with the Z axis passing through the centre of the earth (or *reference ellipsoid*) and the poles, the X axis through the centre and the

Greenwich meridian, and the Y axis at right angles to these, see **Diagram 4**. Other parameters may define this system, but are not directly relevant here.

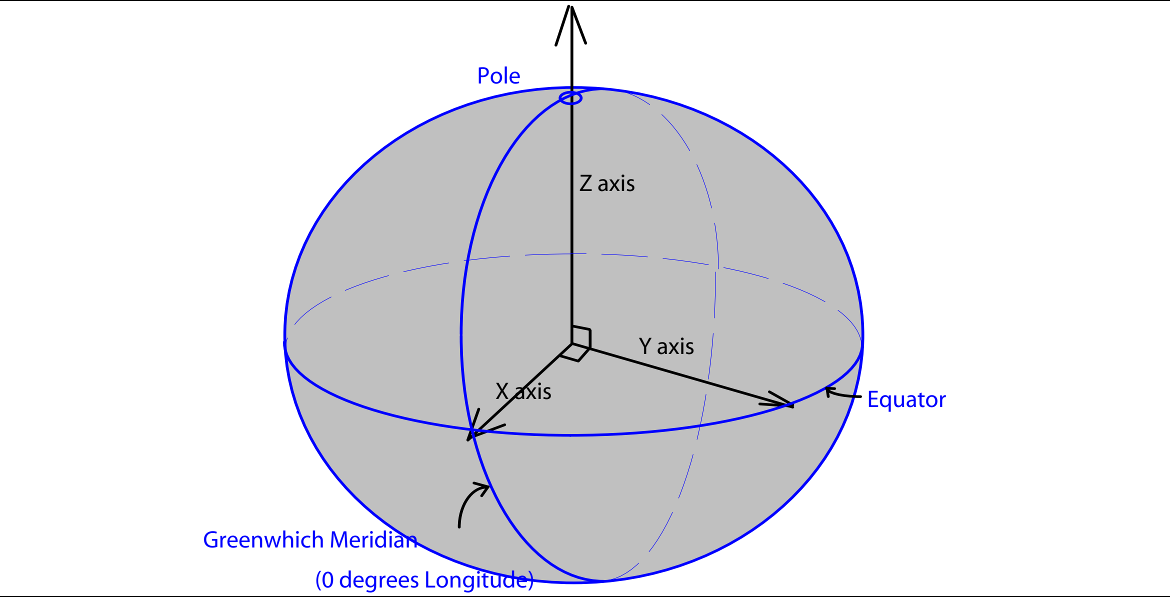


Diagram 4 : *Cartesian Reference System.*

It is important to realise that the centre of a Cartesian Reference System for an adopted *ellipsoid* may not be the 'true' or adopted centre of mass of the earth - the latter is often used for global reference systems, such as the geocentric Global Positioning System, (GPS). Furthermore, the direction of the Z axis may differ. This has led to one of the major requirements of geodesy and surveying today, which is how to relate global to local referencing systems so that positions in one may be expressed in terms of the other and vice-versa. This issue is dealt with in the joint technical publication **[10]**, which documents the necessity to move from the ellipsoidal system to the local Cartesian Reference System and vice-versa.

The Cartesian Reference System is particularly useful as calculations are simpler to perform because no knowledge of spherical geometry is required. However, the relationship between positions on the earth's surface are difficult to visualise in this system, and the concept of height is also unclear.

2.9 Plane Co-ordinates

Having established a co-ordinate reference system, it remains to depict the position of points of interest onto a flat surface, e.g. a piece of paper. In general, it is usual to determine the position of some reference points in terms of the *ellipsoidal reference system* by measurement, such as a geodetic triangulation network, before projecting these onto a plane co-ordinate system and carrying out the survey of topographical detail in this simpler system. Projecting co-ordinates from a curved surface, in latitude (ϕ) and longitude (λ), onto a plane surface will cause some distortion, but this is minimised by choosing the most appropriate projection for the area concerned, and the features to be depicted.

Numerous projections can be used, many of which may be seen in any world atlas. However, because the areas to be projected in a world atlas are large the distortions are unavoidable and clearly evident. Engineering and cadastral maps are generally of smaller areas depicted at scales larger than 1:50 000, and as these forms of map are often the basis for further measurement, it is necessary to keep distortions to a minimum. The property of retaining shape and scale is known as *orthomorphism*, or *conformality*.

2.10 The Transverse Mercator Projection

One projection with the properties of *orthomorphism* is known as the *Transverse Mercator,* or *Gauss Conformal* projection. Although conceptually simple, the mathematics involved are not so, if orthomorphism is to be achieved. The basic formulae are given in **Section 4** of this paper.

A cylinder of a specified radius is wrapped around the reference *ellipsoid* so that its circumference touches the *ellipsoid* along a chosen meridian or line of longitude, see **Diagram 5**. The scale of the projected area is therefore correct along the chosen meridian. The radius of the cylinder will match the radius of the *reference ellipsoid* at a specified point. This provides the o*rigin* of the projection i.e. the line of longitude at which the cylinder makes contact, and the latitude of the *reference ellipsoid* where the radii match. The cylinder is then 'unwrapped', providing the flat surface of the map. The *origin* is generally chosen to be central to the area of interest, so that the distortions away from the *origin* are minimised.

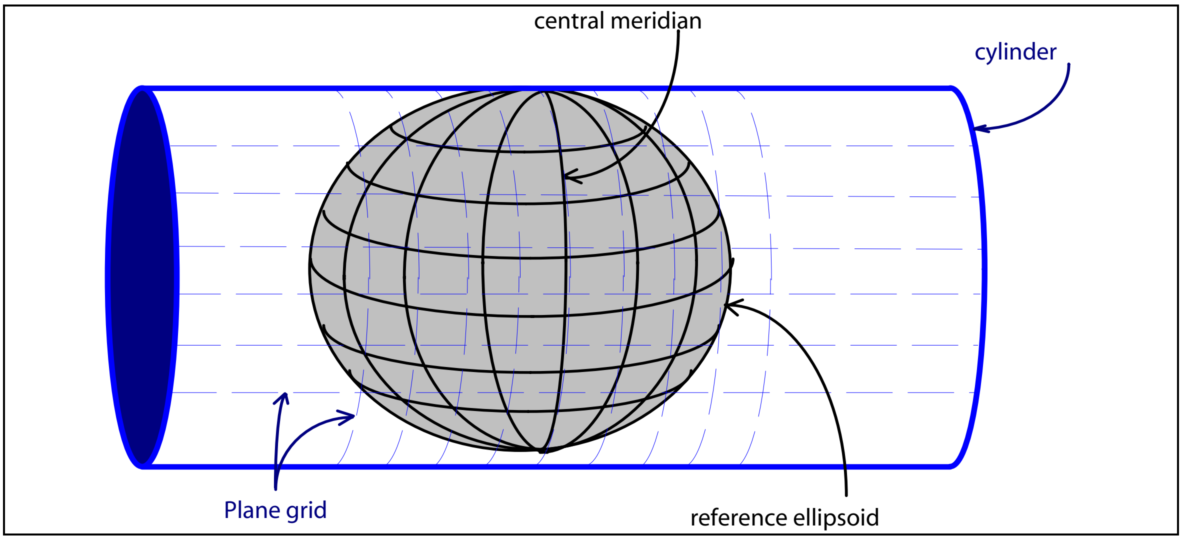


Diagram 5 ***: Concept of Transverse Mercator Projection***

Scale increases away from the *origin* in a uniform way, so that scale at any point on the grid away from the central meridian (where it is uniform) may be corrected by applying a *scale factor* to determine true scale[2]. In this way, for short lines generally less than 100 metres, true ground distances may be obtained by measurement off the map.

[2] Implementations of the Transverse Mercator Projection can vary, for example, in Great Britain the scale factor of the central meridian is reduced from 1.0 to 0.9996 approximately, reducing the size of the scale factor to be applied at the extremes of the area mapped.

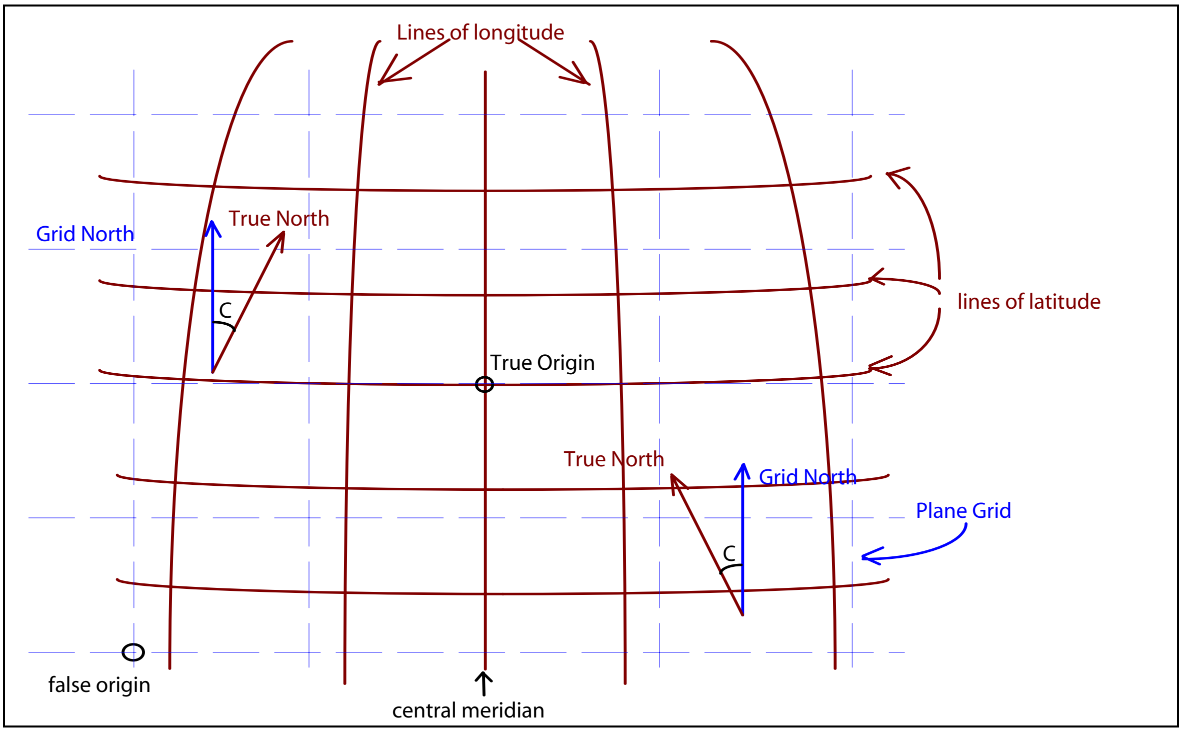


Diagram 6 : *Lines of latitude and longitude projected onto the plane grid reference system using the Transverse Mercator Projection, and Convergence, (C).*

In order to avoid negative co-ordinates within the area of interest because the *origin* will have plane co-ordinates of 0 in eastings and 0 in northings, it is usual to add a constant value to eastings and northings so that all plane co-ordinates are positive. This creates a *false origin* for the projection.

As lines on the *reference ellipsoid* project onto the plane grid as curved lines, two further corrections are required for precise computations. Along the central Meridian true north (the direction of the north pole) and grid north are the same. However, away from the central meridian the direction of true north differs by an amount which increases the further from the central meridian the point of interest is situated. This difference is known as *Convergence (C)*, and it is the angle between true north and grid north, see **Diagram 6**.

Straight lines on the *reference ellipsoid* project on to the plane grid as curved lines, known in geodesy as *geodesics*, in order to compensate for this effect, a further correction to a direction at a point is required. This is known as the *Arc to chord*, or *(t-T) correction* and it is the difference in angle between the initial direction of the curved *geodesic* and the straight line grid bearing. *Geodesics* are always concave to the central meridian.

Diagram 7, is greatly exaggerated to show two directions either side of the central meridian. The straight line A-B represents a line between two points as plotted on the map, whose direction is termed *t*. The curved line A-B represents the line of sight between the same two points in nature which projects onto the map as a curved line, the *geodesic*, whose initial direction is termed *T*. The direction of the curved line A-B at A is represented by the line A-X, and it's initial direction is the tangent of the curve at A.

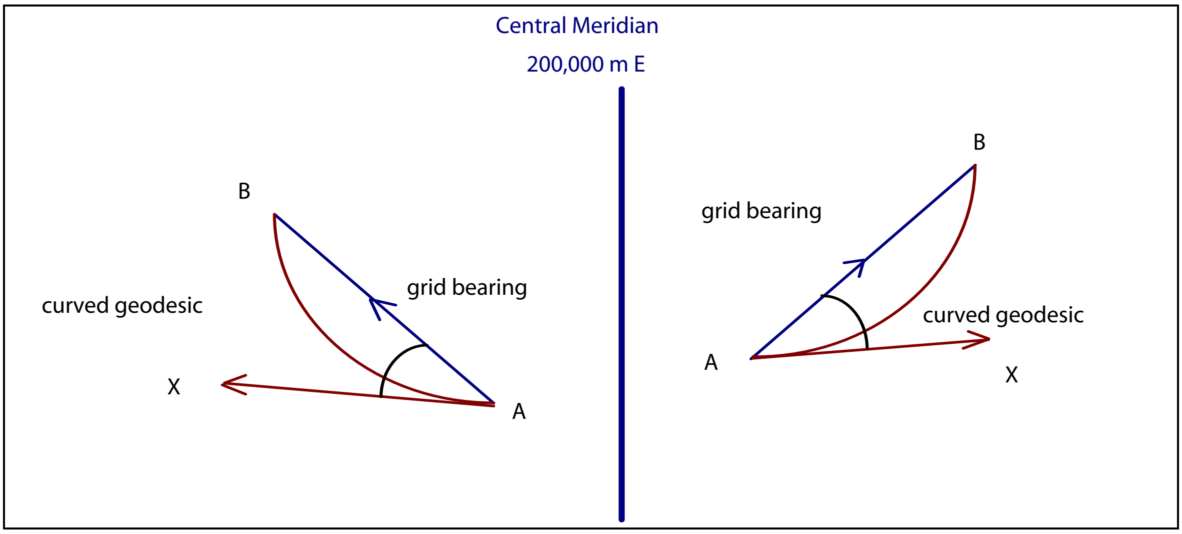


Diagram 7*: The (t-T) correction*

It should be noted that the Transverse Mercator projection is *conformal*, therefore directions on the projection are relatively correct, and this correction is required only when comparisons with the real world are necessary, and only if one is interested in a few seconds of arc.

3.0 Historical Context

3.1 Introduction

The Irish Grid has developed over more than two hundred years, in line with the development of scientific thought, measurement techniques and computational power. In recent times the development and wide use of new technologies, such as GPS and increasingly powerful computers, has highlighted some of the shortcomings of reference systems in use all over the world. This section provides a summary of the historical development of the Irish Grid to date. A fuller account may be obtained from the references given in **Section 7**.

3.2 The Principal Triangulation of Great Britain and Ireland

The Principal Triangulation was begun in 1783, to determine the difference in longitude between the observatories of Greenwich and Paris. Following the establishment of the Ordnance Survey in 1791 it was gradually extended to cover the whole of Great Britain and Ireland, with observations completed by 1853. The triangulation approach adopted was necessary in order to transfer distances from the measurement of a base line on Salisbury Plain across the length and breadth of Great Britain and Ireland.

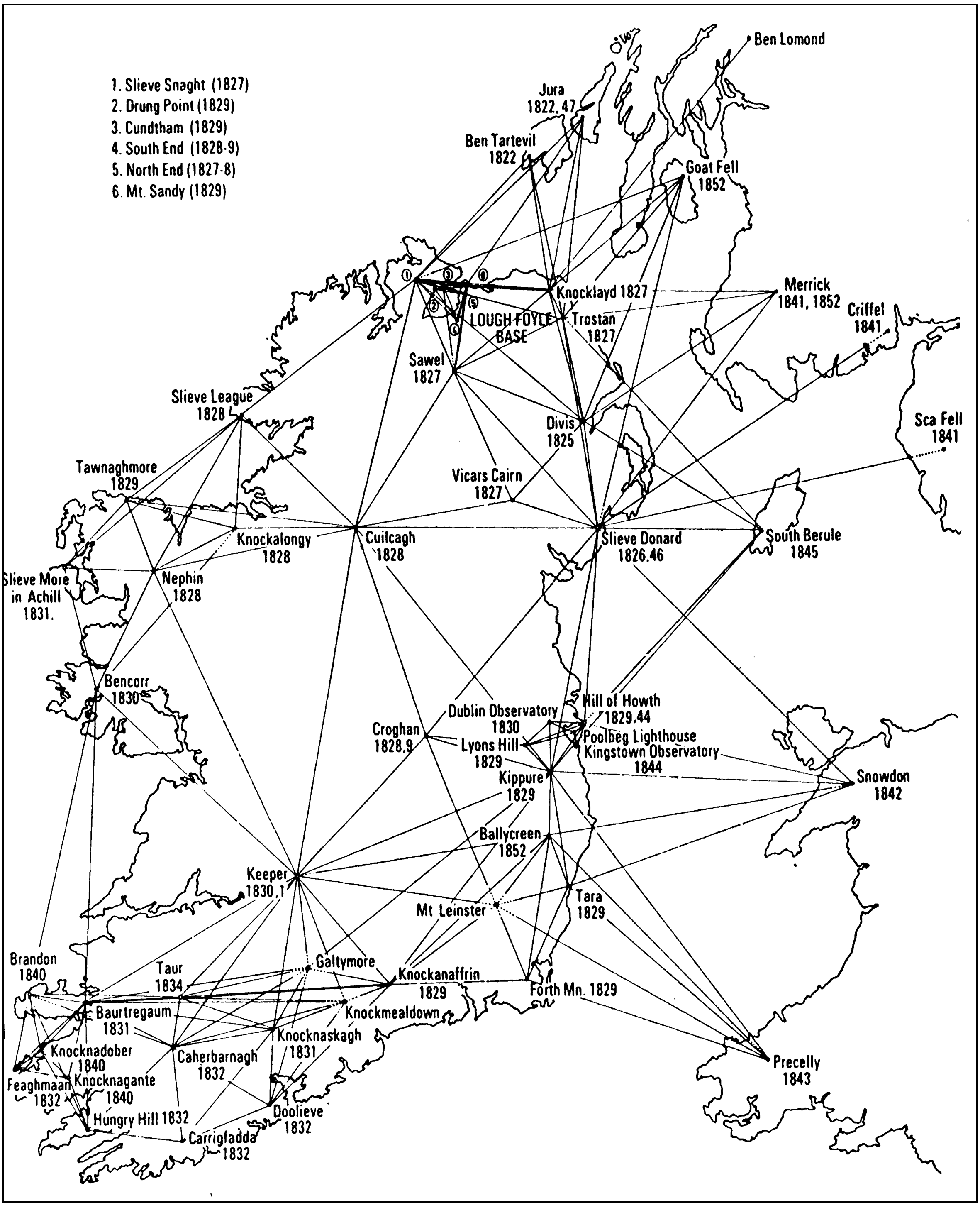


Diagram 8 ***: The Principal Triangulation of Ireland 1824 - 1832***

In 1824 the Spring Rice Committee recommended to the British House of Commons that a survey of Ireland at the scale of six inches to the mile was required to provide a definitive indication of acreages and rateable values for the purposes of establishing local taxes in Ireland. This task was begun by the Ordnance Survey on 22 June 1824.

The Principal Triangulation in Ireland commenced shortly after the Spring Rice Committee, and was completed by August 1832. It was not until 1858, however, that A.R. Clarke, who was then in charge of the Trigonometrical and Levelling Department of Ordnance Survey, had selected the sufficient observations to form the interlocking network of well conditioned triangles that is now known as the Principal Triangulation, see **Diagram 8**. A.R.Clarke, rigorously adjusted the network by the method of least squares in 21 independently computed, but connected blocks for Great Britain and Ireland, reference [**5**].

Bearing in mind the early date of many of the observations and the primitive nature of the instruments used, the results from the Principal Triangulation were impressive. The average of the triangular misclosures was 2.8 seconds of arc, and the distance of the Lough Foyle base as computed through the triangles from the Salisbury Plain base was within 5 inches of it’s measured length, reference [**6**].

3.3 Early Map Control for Ireland

The Principal Triangulation was not designed as a comprehensive national system to control mapping, but as a geodetic framework initially to determine the difference in longitude between Paris and London and later to define a figure for the earth. The results of the Principal Triangulation adjustment were not available until the late 1850’s, reference [**5**].

Six Inch Map Control
Some form of framework was, however, required on which to control the new mapping of Ireland at the scale of six inches to one mile. Six inch map control was therefore based on a network of secondary and tertiary ‘blocks’ of triangulation, begun in 1832 and completed in 1841, just ahead of the chain survey teams who were surveying the detail. Although these lower order triangulations may have included some of the Principal Triangulation points they were probably based on provisional values only. Furthermore, the blocks were computed independently of each other and by a variety of methods. This was expedient at the time, their purpose being for mapping on a county by county basis only. However, the result was that little sympathy existed between adjacent blocks of map control, which together with spherical reference systems used, independent county meridians and their associated Cassini projections caused apparent discrepancies of up to 50 ft. between detail across adjoining counties, reference [**6**].

One Inch Map Control
The one inch map, begun in 1852, was to be on a single national datum and projection, and compiled from the detail surveyed for the six-inch maps. Again the results from Clarke’s adjustment were not available, and the method chosen to relate the county datums to the national one did not fully connect to the Principal Triangulation, reference [**5**]; there were only five common points between it and the 32 county datums.

25 Inch Map Control
October 1887 saw Treasury approval granted for the 1:2 500 scale mapping of Ireland, and the work began in 1888. Initial methods involved replotting from the six-inch field books at the larger scale, but this was subsequently abandoned following the completion of Counties Down and Limerick in favour of a resurvey based on new secondary and tertiary

triangulation. However, as before, secondary blocks were adjusted independently of each other and tertiaries by semi-graphical methods and although Clarke's adjustment was now complete, it was not used, and few records survive today of the work undertaken. The task was largely completed by 1913, and the results thus derived remain the basis of large scale maps for many parts of Ireland today.

Whatever may be said today about the methods of control chosen, they were adequate for the purposes they served at that time, and, *'When the map of Ireland is picked up and shaken, it is only the mathematician who hears the rattle'* (Andrews, see reference [**7**] p.233). This 'rattle', however, is of more relevance in today's age of electronic computers, global geodetic frameworks and mapping across Europe than it was in the 1850's, therefore in recent times a number of actions have been taken to improve the situation.

3.4 The Re-Triangulation of Northern Ireland

After the Ordnance Survey of Northern Ireland (OSNI) was set up (1921), the Ordnance Survey of Great Britain (OSGB) retained responsibility for the geodetic triangulation until the end of the second World War. Priorities clearly lay elsewhere between and during the World Wars, and economic conditions did not help, therefore little action was taken to improve the condition of the geodetic framework or the mapping until after World War II.

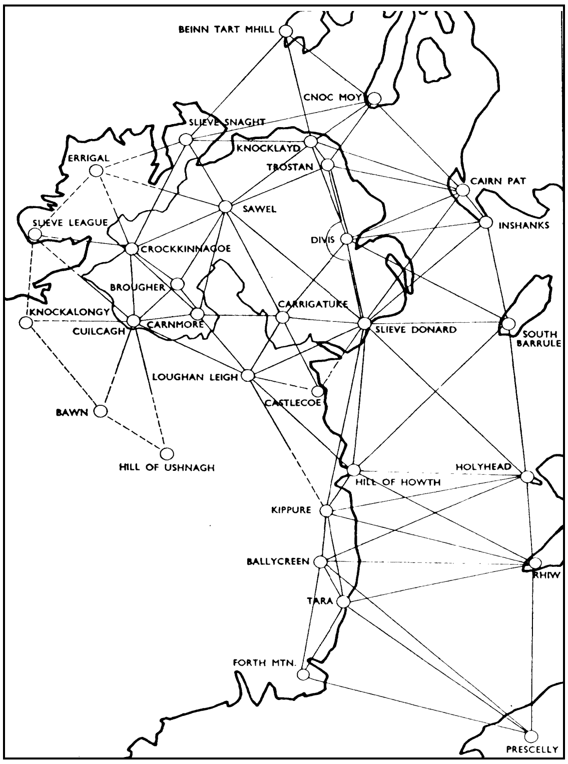


Diagram 9 ***: The Re-Triangulation of Northern Ireland 1952***

Following the completion of the re-triangulation of Great Britain, reference [**6**] (itself delayed due to World War II), resources were made available to OSNI and observations on the re-triangulation began in the Spring of 1952. The network consisted of 9 stations, plus 3 in the Republic of Ireland and a number of cross-channel connections to Great Britain, see **Diagram 9**.

The adjustment accepted the position of three of the original Principal Triangulation points in order to scale and orientate the new triangulation. One of these points was fixed to Clarke's original value (at Divis) and two (Knocklayd and Trostan) to values from Wolf's 'Mathematical Basis' (unknown reference) which varied slightly from Clarke's, see reference [**8**]. Comparisons with the (then) recent re-triangulation of Great Britain showed only a slight discrepancy between the two re-triangulations.

3.5 Mapping Control in Northern Ireland.

As soon as the Primary Triangulation was completed the Secondary Triangulation was re-commenced (one block having been completed earlier). This work finished in July 1956, and each secondary block was adjusted separately to the primary and to adjoining secondary stations.

The Secondary Triangulation was further broken down into Tertiary Triangulation and connections incorporated to stations used for controlling the 25 inch mapping. This established the differences between the county-based datums, and the Irish Grid, allowing the mapping to be 're-cast' on to the Irish Grid, within tolerances acceptable at the time, reference [**8**]. These networks also formed the basis for re-surveys of some areas of mapping which were either too out of date or where significant accuracy problems were known to exist.

3.6 The Primary Triangulation of Ireland

The Ordnance Survey of Ireland (OSi) carried out a first order triangulation of Ireland between the summer of 1962 and late 1964, the whole island being adjusted as one, with the observations from the 1952 Re-Triangulation of Northern Ireland included, see **Diagram 10**. At the time, the availability of new Electromagnetic Distance Measurement (EDM) equipment allowed distances over long lines, say 30km to 50km, to be determined quickly for the first time. This was achieved by measuring the sides of a braced quadrilateral figure by Tellurometer EDM in the south west of Ireland, providing some independent scale checks on the triangulation network.

Some constraints on the new scheme were necessary. The new adjustment had to restrict the movement of the northern primaries to within 0.25 m of their 1952 positions, a tolerance chosen so as not to affect the mapping already completed or recast on to the Irish Grid in Northern Ireland. Initially the positions of Slieve Donard and Cuilcagh, see **Diagram 10**, were held fixed to their values from the 1952 adjustment, with tellurometer and cross channel observations excluded. This caused too large a movement of the Northern Ireland stations from their 1952 values (up to 1.2 m in one instance), probably because the 1952 values were originally based on the Principal Triangulation 1824-1832, which contained an inherent scale error of between 30 and 40 parts per million (ppm). A modified *ellipsoid* and projection was, therefore, tried and found to improve matters, moving Northern Ireland stations by 0.5m from their 1952 values, but causing unacceptable shifts at the common stations in the south.

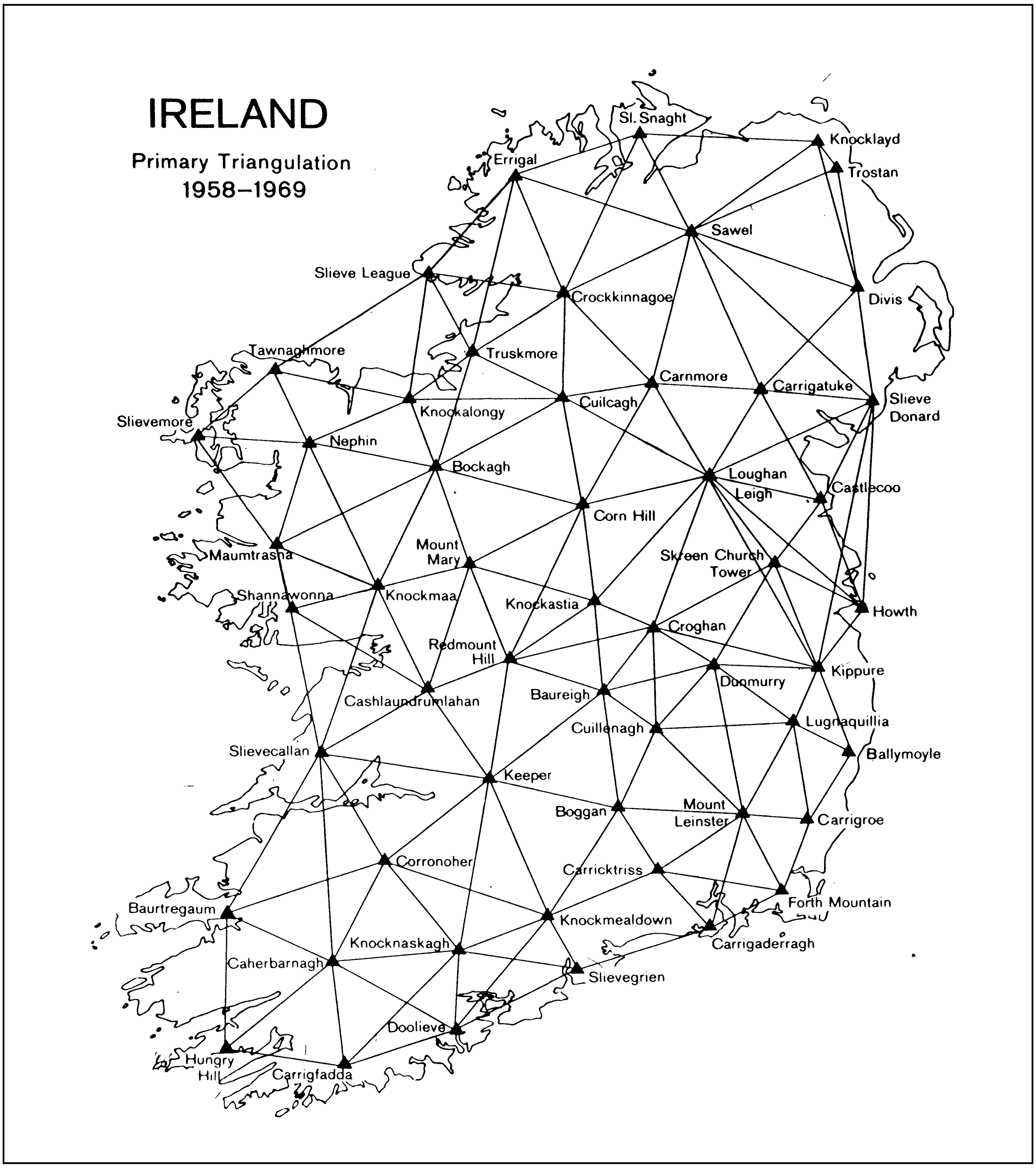


Diagram 10 ***: The Primary Triangulation of Ireland.***

Thus the Airy *ellipsoid* was modified by 35 ppm, and a modified Transverse Mercator projection adopted using the Airy Modified Ellipsoid. This gave an overall scale correction to conform to the tellurometer distances observed while retaining geographical and projection co-ordinates substantially unaltered, with the grid to graticule tables, used for conversion between the two, remaining completely unaltered. This latter fact was of course very important before the days of electronic calculators and computers.

The results from this adjustment are summarised in the next table:

Root Mean Square correction to an observed angle	± 0".96
Maximum adjustment correction	2".18
Average angle correction due to scale factor	0".002
Root Mean Square Error (RMSE) of observed *v* computed distances	± 7 ppm
Maximum RMSE of observed vs computed distances	±12 ppm

A comparison with the 1952 adjustment gave root mean square changes of ± 0.25 m in Eastings and ± 0.23 m in Northings and in a maximum vector difference of 0.57 m.

Subsequent *Laplace* observations were carried out on two lines (Carnmore to Carrigatuke and Doolieve to Carrigfadda) in 1966, and thus were not included in the 1965 adjustment. These indicated, however, that a mean rotation of the network by +2.27 seconds would satisfy the *Laplace* conditions. Comparison of the two lines showed only 0.5 seconds of arc difference between them, indicating very little internal distortion in the network, albeit a small rotation overall.

Further work in 1969 re-observed Tellurometer EDM distances across St. George's Channel and the Irish Sea, and observed Geodimeter EDM distances between four further Irish stations. These latter observations revealed a continuing scale error of +5ppm in the 1965 adjustment.

3.7 1975 Mapping Adjustment

OSNI did not adopt the 1965 values for subsequent mapping, whereas OSi did. To address the problem of large scale maps meeting at the border a re-computation of the 1952 and 1962 observations was begun. However, many parts of Ireland had already been mapped, therefore a major restraint on this new adjustment was to prevent any plottable shift in those areas already mapped. (i.e. Northern Ireland, Dublin and Cork). Thus all Northern Ireland primary stations were held fixed at their 1952 values and three primaries in the Republic (Howth, Kippure, and Doolieve) were fixed at their 1965 values. Only angle observations were used in the adjustment, which is known as the 1975 Mapping Adjustment, and the resulting shift from the 1965 adjustment was as follows:

Average difference in Eastings	0.092 m
Average difference in Northings	0.108 m
Maximum vector difference	0.548 m

The Airy Modified Ellipsoid was again used, and the datum (known as the 1965 datum) is derived from the values of the stations held fixed in the adjustment.

3.8 IRENET'95

During the late 1980's and early 1990's the increased use of GPS for mapping and scientific work highlighted the need for a reference network compatible with the new technology and essential to relate the mapping framework to global and continental reference systems. In April 1995 a new geodetic and survey control network (IRENET) was observed using the Global Positioning System (GPS). This network consists of 12 new 'zero-order' control stations (8 in the Republic of Ireland, 3 in Northern Ireland and one on the Isle of Man) connected to some of the defining International Terrestrial Reference Frame (ITRF) stations in Europe, see **Diagram 11**. The resulting adjustment was accepted as an official extension to the European Terrestrial Reference System (ETRS) by sub-commission X (EUREF) of the International Association of Geodesy (IAG) in Ankara, Turkey, 1996, reference [**9**].

These zero order stations were used to control a densification of the network to a further 173 stations throughout Ireland between May and December of the same year. Co-ordinates have been computed in terms of ETRS89 and Irish Grid (1975 Mapping Adjustment), and these stations will form the basis for all future scientific and mapping control work undertaken by OSi and OSNI.

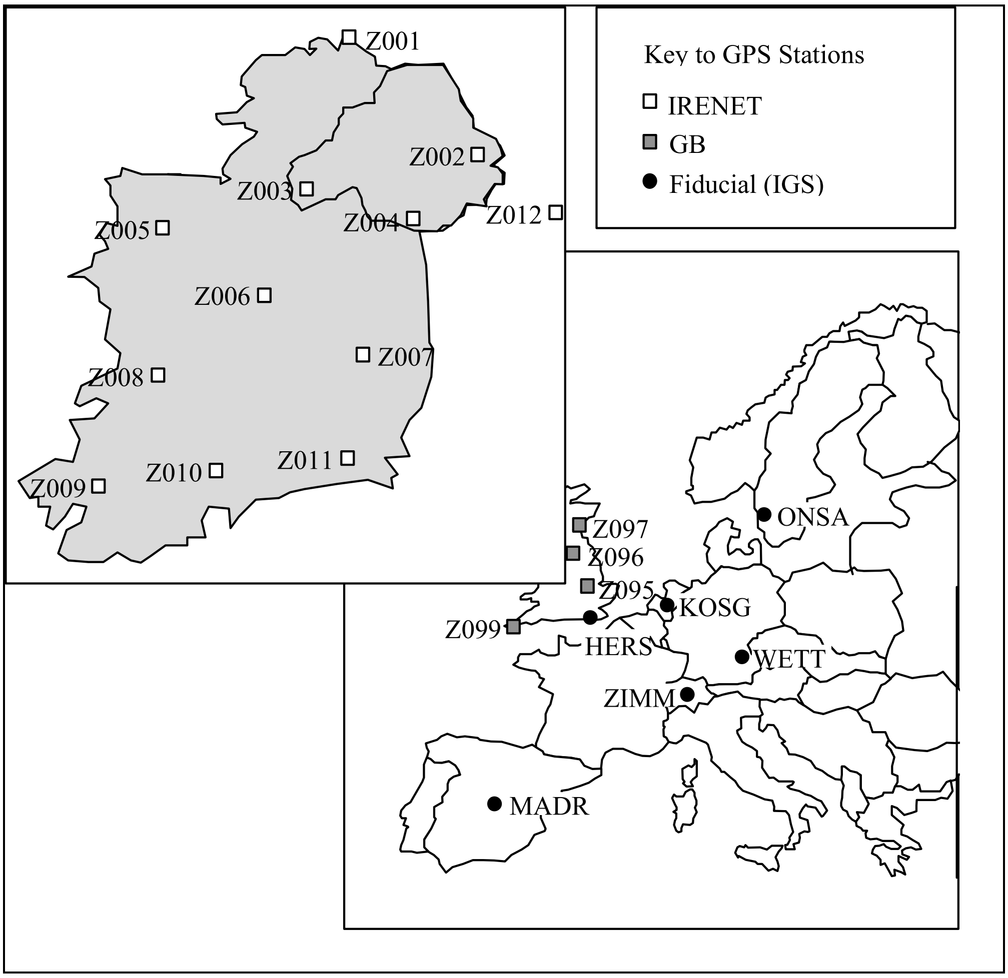


Diagram 11 ***: IRENET Zero Order Stations.***

4.0 Technical Data

4.1 Overview

National Reference System	Irish Grid
Reference Ellipsoid	Airy Modified
Geodetic Datum	1965 Datum
Vertical Datum	Malin Head and Belfast
Map Projection	Transverse Mercator
Measurement Unit	International metre

4.2 Definitions and Parameters

Airy Modified Ellipsoid
All geographical co-ordinates, on which the Irish Grid is based, are expressed in terms of the Airy Modified Ellipsoid, as fixed onto the 1965 Datum. This ellipsoid is based on the Airy Ellipsoid, defined in feet of bar O_1 [3], with a semi-major axis (a) of 20,923,713 feet, and eccentricity squared (e^2) of 0.006 670 540 15. With metrication a conversion factor was agreed between feet of bar O_1 to the International metre of 0.304 800 749 1. The Airy Ellipsoid was reduced by 35 parts per million (ppm) for the Irish reference ellipsoid, resulting in the following, standard parameters:

semi-major axis (a) :	6 377 340.189 m
eccentricity (e^2) :	0.006 670 540 15

The 1965 Geodetic Datum
The Geodetic Datum of the Irish Grid is a derived one based on the positions of nine OSNI primary triangulation stations (1952 adjustment values), and the positions of three OSi primary triangulation stations fixed to their 1965 adjustment values. The 1965 adjustment was a best mean fit to the positions of the Northern Ireland Primary points as adjusted in 1952. The 1952 adjustment was based on the Principal Triangulation positions of three points in Northern Ireland: Knocklayd, Trostan and Divis, see **Diagram 9**.

[3] a bar of standard length kept by Ordnance Survey Great Britain, Southhampton.

Vertical Datum

Currently, there are three Vertical Datums in Ireland. One was fixed as Mean Seal Level (MSL) of the tide gauge at Malin Head, County Donegal established in 1957. Another is MSL of a tide guage established at Clarendon Dock, Belfast in the 1950's. The third was used to depict heights, in Imperial feet, on earlier maps (e.g. County Series) and used the low water mark of the spring tide on the 8 April 1837 at Poolbeg Lighthouse, Dublin.

Malin Head was adopted by OSi as a datum in 1970 from readings taken between January 1960 and December 1969. All heights on Irish Grid mapping produced by OSi since then are in International metres above this datum. Since this Vertical Datum is only 0.037 metres above MSL Belfast, OSNI adopted this datum for the new all Ireland 1:50,000 map series and other small scale products.

All heights shown on OSNI large scale mapping (1:1,250, 1:2,500 and 1:10,000) are related to the MSL Belfast datum derived from readings on the tide gauge over six years between 1951-56.

Both the Malin Head and Belfast datum's are approximately 2.7 m above the Poolbeg Lighthouse datum and all conversions are subject to localised anomalies.

4.3 The Transverse Mercator Map Projection

Ellipsoid	Airy Modified
True Origin	Latitude 53^o 30' 00" N
	Longitude 8^o 00' 00" W
False Origin	200 kms west of true origin
	250 kms south of true origin
Plane Co-ordinates of True Origin	200 000 E
	250 000 N
Scale Factor on Central Meridian	1.000 035

This projection is used on all Irish Grid Maps at the scales of 1:1 000, 1:1,250, 1:2 500, 1:10,000, 1:50 000 and 1:250 000.

Other co-ordinate systems and projections used in Ireland are listed in **Appendix A**.

5.0 Notation, Symbols and Standard Formulae

All distances are in metres: Conversion feet to metres: 1 ft = 0.304 800 749 1 m

All angles are in radians: Conversion degrees (decimal) to radians $1^{\circ} = \frac{\pi}{180}$, or 0.017 453 293 radians

All constants relate to the Irish Grid and Reference Ellipsoid

5.1 Standard Notations and Definitions

Notation	Description, Formulae and Constants
a	Semi-major axis of ellipsoid. Constant, a = 6 377 340.189 m
b	Semi-minor axis of ellipsoid, Calculated, $b = \sqrt{a^2\left(1-e^2\right)}$ = 6 356 034.447 m
e^2	Eccentricity squared. Constant, e^2 = 0.006 670 540 15
n	$n = \frac{a-b}{a+b}$
ν	Radius of curvature of the ellipsoid at latitude ϕ perpendicular to a merdian (east - west) $\nu = \frac{a}{(1-e^2\sin^2\phi)^{1/2}}$
ρ	Radius of curvature of the ellipsoid at latitude ϕ in the direction of the meridian (north - south) $\rho = \frac{a(1-e^2)}{(1-e^2\sin^2\phi)^{3/2}}$
η^2	East - west component of the deviation of the vertical, squared $\eta^2 = \frac{\nu}{\rho} - 1$
ϕ	Latitude of a point. Can be calculated from E and N of a given point (see page 26).
λ	Longitude of a point (positive (+) east of Greenwich and negative (-) west of Greenwich). Can be calculated from E and N of a given point (see page 26).

h	Height of a point above the ellipsoid. It is necessary to know the geoid - ellipsoid separation at a point, N, so that : $h = (H + N)$ The Airy modified ellipsoid is a good fit to the geoid in Ireland so N can be ignored for most practical purposes, or assumed to be 2.5 m. Use of a geoid model is essential for more precise computations.
H	Height of a point above the geoid (MSL). Generally observed by spirit levelling.
ϕ'	Latitude of the foot of the perpendicular drawn from a point on the projection to the central meridian. An iterative process is used to obtain ϕ' as follows: 1. Calculate initial $\phi' = \left(\frac{N - N_0}{aF_0}\right) + \phi_0$ 2. Calculate M (see below). 3. Calculate new value for ϕ' as follows: $\phi'(\text{new}) = \left(\frac{N - N_0 - M}{aF_0}\right) + \phi'_{old}$ 4. Recalculate M using ϕ'(new) in place of ϕ'(initial). 5. If $(N - N_0 - M)$ is close to zero (i.e. < 0.001) then use ϕ'(new). Otherwise recalculate M using ϕ'(new), and repeat steps 3 and 4 above.
ϕ_0	Latitude of the True Origin. Constant, $\phi_0 = 53^o\ 30'\ 00''$ N
λ_0	Longitude of the True Origin. Constant, $\lambda_0 = 8^o\ 00'\ 00''$ W
E_0	Grid Easting of the True Origin. Constant, E_0 = 200 000 E
N_0	Grid Northings of the True Origin. Constant, N_0 = 250 000 N
E	Grid Eastings of a point. Can be calculated from ϕ and λ (see page 25).
N	Grid Northings of a point. Can be calculated from ϕ and λ (see page 25).
y	'True' Eastings of a point $y = E - E_0$
x	'True' Northings of a point $x = N - N_0$
F_0	Scale Factor on the Central Meridian. Constant, F_0 = 1.000 035
F	Scale Factor at a point. Can be calculated from E and N or from ϕ and λ (see page 27).

S	True distance between two points on the ellipsoid $S = \frac{s}{F}$
s	Straight line distance between two points on the projection s = S x F
A	True meridional arc A = b [(i)-(ii)+(iii)-(iv)] b has been scaled by F_o Where $(i) = \left\{\left(1 + n + \frac{5}{4}n^2 + \frac{5}{4}n^3\right)(\phi - \phi_0)\right\}$ $(ii) = \left\{\left(3n + 3n^2 + \frac{21}{8}n^3\right)\sin(\phi - \phi_0)\cos(\phi + \phi_0)\right\}$ $(iii) = \left\{\left(\frac{15}{8}n^2 + \frac{15}{8}n^3\right)\sin 2(\phi - \phi_0)\cos 2(\phi + \phi_0)\right\}$ $(iv) = \left\{\frac{35}{24}n^3 \sin 3(\phi - \phi_0)\cos 3(\phi + \phi_0)\right\}$
M	Developed Meridional arc M = A x F_0
C	Convergence - the angle between the grid north and true north at a point. Can be calculated from either ϕ and λ or E and N (see page 28).
t	Straight line direction (or chord) between two points on the projection (see page 28).
T	Direction of the line of sight between two points on the ellipsoid as it appears on the projection (direction of the geodesic), (see page 28).
P	Difference in longitude between a point and the longitude of the True Origin (in radians) $P = \lambda - \lambda_0$

5.2 Computation of Eastings and Northings (E and N) from Latitude and Longitude (ϕ and λ)

$$\mathbf{N} = (I) + P^2(II) + P^4(III) + P^6(IV)$$

where

$$P = (\lambda - \lambda_0)$$

and :

$$(I) = M + N_0$$

$$(II) = \left(\frac{\nu}{2}\sin\phi\cos\phi\right)$$

$$(III) = \left(\frac{\nu}{24}\sin\phi\cos^3\phi\left(5 - \tan^2\phi + 9\eta^2\right)\right)$$

$$(IV) = \left(\frac{\nu}{720}\sin\phi\cos^5\phi\left(61 - 58\tan^2\phi + \tan^4\phi\right)\right)$$

For N_0, F_0, A and other elements - see **Standard Formulae** above.

NB: a and b have been scaled by F_0.

$$\mathbf{E} = E_0 + F_0\left[P(V) + P^3(VI) + P^5(VII)\right]$$

where:

$$P = (\lambda - \lambda_0)$$

and

$$(V) = (\nu\cos\phi)$$

$$(VI) = \left(\frac{\nu}{6}\cos^3\phi\left(\frac{\nu}{\rho} - \tan^2\phi\right)\right)$$

$$(VII) = \left(\frac{\nu}{120}\cos^5\phi\left(5 - 18\tan^2\phi + \tan^4\phi + 14\eta^2 - 58\tan^2\phi\eta^2\right)\right)$$

For E_0,F_0 and other elements- see **Standard Formulae** above.

NB: a and b have been scaled by F_0.

5.3 Computation of Latitude and Longitude (ϕ and λ) from Eastings and Northings (E and N)

$$\phi = \phi' - y^2(VIII) + y^4(IX) - y^6(X)$$

where

$$(VIII) = \left(\frac{\tan\phi'}{2\rho\nu}\right)$$

$$(IX) = \left(\frac{\tan\phi'}{24\rho\nu^3}\left(5 + 3\tan^2\phi' + \eta^2 - 9\tan^2\phi'\eta^2\right)\right)$$

$$(X) = \left(\frac{\tan\phi'}{720\rho\nu^5}\left(61 + 90\tan^2\phi' + 45\tan^4\phi'\right)\right)$$

For ϕ‘ and other elements see **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

$$\lambda = \lambda_0 + y(XI) - y^3(XII) + y^5(XIII) - y^7(XIV)$$

where

$$(XI) = \left(\frac{\sec\phi'}{\nu}\right)$$

$$(XII) = \left(\frac{\sec\phi'}{6\nu^3}\left(\frac{\nu}{\rho} + 2\tan^2\phi'\right)\right)$$

$$(XIII) = \left(\frac{\sec\phi'}{120\nu^5}\left(5 + 28\tan^2\phi' + 24\tan^4\phi'\right)\right)$$

$$(XIV) = \left(\frac{\sec\phi'}{5040\nu^7}\left(61 + 662\tan^2\phi' + 1320\tan^4\phi' + 720\tan^6\phi'\right)\right)$$

For ϕ‘ and other elements see **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

5.4 Computation of True Distance and Grid Distance

The relationship between a distance on a map and the true (natural) distance is expressed as $s = S \times F$, or $S = \frac{s}{F}$,

where S is the true distance, s is the grid (map) distance and F is the scale factor. Calculating the scale factor of the mid point of a line is sufficiently accurate for most purposes.

For greater accuracy the scale factor at the start point, mid point and end point of a line should be calculated and Simpson's rule applied:

$$\frac{1}{F} = \frac{1}{6}\left(\frac{1}{F_{start}} + \frac{4}{F_m} + \frac{1}{F_{end}}\right)$$

Scale Factor at a Point from φ and λ

$$\mathbf{F} = \boldsymbol{F_0\left[1 + P^2(XV) + P^4(XVI)\right]}$$

where:

$$(XV) = \frac{\cos^2\phi}{2}\left(1+\eta^2\right)$$

$$(XVI) = \frac{\cos^4\phi}{24}\left(5 - 4\tan^2\phi + 14\eta^2 - 28\tan^2\phi\eta^2\right)$$

P, η and ν are calculated from the **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

Scale Factor at a point from E and N

$$\mathbf{F} = \boldsymbol{F_0\left[1 + y^2(XVII) + y^4(XVIII)\right]}$$

where:

$$(XVII) = \frac{1}{2\rho\nu}$$

$$(XVIII) = \frac{1+4\eta^2}{24\rho^2\nu^2}$$

For φ' and other elements see **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

5.5 True Azimuth from Grid Co-ordinates

For a given line A to B :

True Azimuth (A to B) = Grid bearing (A to B) + C - (t-T)

where C is the convergence and (t-T) is the arc to chord correction. The signs (positive or negative) are important, and it is recommended that a diagram be drawn to ensure the corrections are applied in the correct sense, see diagram and example below.

Convergence C from ϕ and λ)

$$C = P(XIX) + P^3 (XX) + P^5 (XXI)$$

where :

$$(XIX) = \sin\phi$$

$$(XX) = \frac{\sin\phi\cos^2\phi}{3}\left(1+3\eta^2+2\eta^4\right)$$

$$(XXI) = \frac{\sin\phi\cos^4\phi}{15}\left(2-\tan^2\phi\right)$$

P and η are calculated from the **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

Convergence C from E and N

$$C = y(XXII) - y^3(XXIII) + y^5(XIV)$$

where :

$$(XXII) = \frac{\tan\phi'}{\nu}$$

$$(XXIII) = \frac{\tan\phi'}{3\nu^3}\left(1+\tan^2\phi'-\eta^2-2\eta^4\right)$$

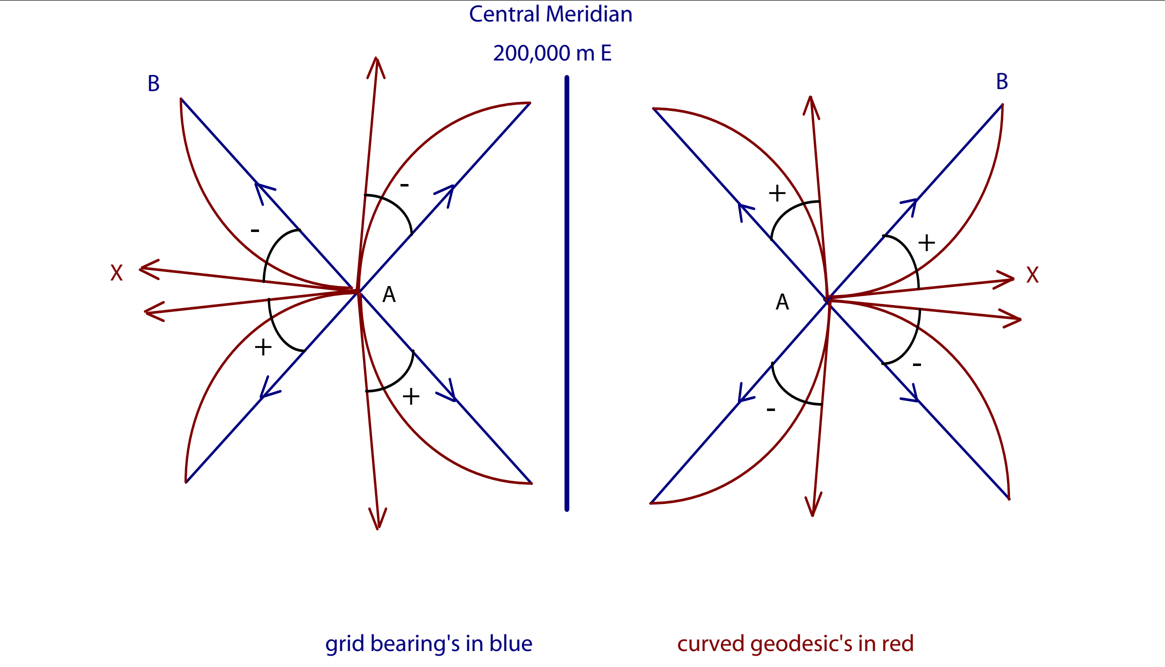
$$(XXIV) = \frac{\tan\phi'}{15\nu^5}\left(2+5\tan^2\phi'+3\tan^4\phi'\right)$$

For ϕ‘ and other elements see **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

(t-T) Correction

As a guide, the following diagram shows geodesic's in each quadrant of the compass, either side of the central meridian, and the signs of the (t-T) correction. The example computation in **Section 6.6** shows a specific case.



The sign of (t-T) correction's

(t -T) from E and N

The following formulae are for correcting real world directions to the plane grid. If 1 and 2 are the terminals of the line, then:

$$(t_1 - T_1) = (2y_1 + y_2)(N_1 - N_2)(XXV)$$

$$(t_2 - T_2) = (2y_2 + y_1)(N_2 - N_1)(XXV)$$

where :

$$(XXV) = \frac{1}{6\rho\nu}$$

For ϕ' use the iterative process described in the Standard Formulae section above, but use the mean value of the Northings at points 1 and 2. Other elements may then be computed as per the **Standard Formulae** above.

NB: a and b have been scaled by F_0 .

Note: The answer is in radians. Convert to seconds multiplying by $\frac{1}{\sin 1''}$

6.0 EXAMPLE COMPUTATIONS

Please Note: In all the following calculations a and b have been scaled by F_0 and may be subject to differences in rounding errors due to an individual users calculation device.

6.1 Example 1 : Computation of Eastings and Northings (E and N) from Latitude and Longitude (ϕ and λ)

a) OSO (Phoenix Park)

Latitude, ϕ = 53^o 21' 50".5441 N
Longitude, λ = 06^o 20' 52."9181 W

(i) = -0.0023769279512547
(ii) = 3.46130364893521E-06
(iii) = 2.07542844095922E-08
(iv) = -3.7575342614459E-11
A = -15130.2334883926

n = 0.00167322034796603
ν = 6391304.32563859
ρ = 6376057.7025783
η^2 = 0.00239123040779954

P = 0.0288322666779889

I = 234869.766511607
II = 1530208.55423369
III = 145902.021809838
IV = -21888.4241245653
V = 3813874.22135243
VI = -182416.601340103
VII = -98720.1961981523

Eastings, E = 309 958.2645
Northings, N = 236 141.9291

b) Howth

Latitude, ϕ = 53^o 22' 23".1566 N
Longitude, λ = 06^o 04' 06".0065 W

(i) = -0.0022185529826205
(ii) = 3.23236200794504E-06
(iii) = 1.93673429493052E-08
(iv) = -3.508539608726E-11
A = -14122.1151195475

n = 0.00167322034796603
ν = 6391307.56716596
ρ = 6376067.40396709
η^2 = 0.00239021362750735

P = 0.0337139118714679

I = 235877.884880453
II = 1530063.82361988
III = 145771.556495997
IV = -21902.7266261809
V = 3813065.2161548
VI = -182570.814483949
VII = -98684.7977252937

Eastings, E = 328 546.3442
Northings, N = 237 617.1863

6.2 Example 2 : Computation of Latitude and Longitude (ϕ and λ) from Eastings and Northings (E and N)

a) OSO (Phoenix Park)

Eastings, E = 309 958.26
Northings, N = 236 141.93

(i) = -0.00217658458592961
(ii) = 3.17165352619869E-06
(iii) = 1.89999091950117E-08
(iv) = -3.4425245878838E-11
A = -13854.9696691283
ϕ' = 53^{o} 22' 31".6984

n = 0.00167322034796603
ν = 6391308.41613419
ρ = 6376069.94479899
η^2 = 0.00238994732917419

VIII = 1.6506139597244E-14
IX = 3.49963747125649E-28
X = 1.02020139910188E-41

XI = 2.62270777207874E-07
XII = 4.94598619656873E-21
XIII = 1.75890124784217E-34
XIV = 7.52067321539881E-48

Latitude, $\phi = 53^o\ 21'\ 50''.5442$ N
Longitude, $\lambda = 06^o\ 20'\ 52''.9183$ W

b) Howth
Eastings, E = 328 546.34
Northings, N = 237 617.19

(i) = -0.00194487640605761
(ii) = 2.83617675824106E-06
(iii) = 1.6972035642991E-08
(iv) = -3.0778062713542E-11
A = -12380.0534863662
ϕ' = $53^o\ 23'19''.4228$

n = 0.00167322034796603
ν = 6391313.15904511
ρ = 6376084.13961744
η^2 = 0.00238845960846801

VIII = 1.65140697291118E-14
IX = 3.50307748554147E-28
X = 1.02190562389051E-41

XI = 2.6235225124939E-07
XII = 4.95126057378679E-21
XIII = 1.76207752818572E-34
XIV = 7.53971553553849E-48

Latitude, $\phi = 53^o\ 22'\ 23''.1567$ N
Longitude, $\lambda = 06^o\ 04'\ 06''.0067$ W

6.3 Example 3 : Computation of Scale Factor from Latitude and Longitude

a) OSO (Phoenix Park)

Latitude, $\phi = 53^o\ 21'\ 50''.5441$ N
Longitude, $\lambda = 06^o\ 20'\ 52.''9181$ W

ν = 6391304.32563859
ρ = 6376057.7025783
η^2 = 0.00239123040779954
(XV) = 0.178468264698499
(XVI) = -0.012261592793434

Scale Factor = 1.000 183 36

b) Howth
Latitude, ϕ = 53° 22' 23".1566 N
Longitude, λ = 06° 04' 06".0065 W

ν = 6391307.56716596
ρ = 6376067.40396709
η^2 = 0.00239021362750735
(XV) = 0.178392196865632
(XVI) = -0.0122766174316604

Scale Factor = 1.000 237 76

6.4 Example 4 : Computation of Scale Factor from Eastings and Northings

a) OSO (Phoenix Park)

Eastings, E = 309 958.26
Northings, N = 236 141.93

(i) = -0.00217658458592961
(ii) = 3.17165352619869E-06
(iii) = 1.89999091950117E-08
(iv) = -3.4425245878838E-11
A = -13854.9696691283
ϕ' = 53° 22' 31".6984
ν = 6391308.41613419
ρ = 6376069.94479899
η^2 = 0.00238994732917419

(XVII) = 1.22695082389524E-14
(XVIII)= 2.53299951778783E-29

Scale Factor = 1.000 183 36

b) Howth
Eastings, E = 328 546.34
Northings, N = 237 617.19

(i) = -0.00194487640605761
(ii) = 2.83617675824106E-06
(iii) = 1.6972035642991E-08
(iv) = -3.0778062713542E-11
A = -12380.0534863662
ϕ' = 53° 23'19".4228

ν = 6391313.15904511
ρ = 6376084.13961744
η^2 = 0.00238845960847

(XVII) = 1.22694718188187E-14
(XVIII)= 2.53296954941824E-29

Scale Factor = 1.000 237 76

6.5 Example 5 : Computation of True Distance from Grid Distance

a) Line OSO (Phoenix Park) to Howth

Grid Distance OSO to Howth = 18 646.53 m

1st point, OSO	Eastings	= 309 958.26 E
	Northings	= 236 141.93 N
2nd point, Howth	Eastings	= 328 546.34 E
	Northings	= 237 617.19 N
mid point	Eastings	= 319 252.30 E
	Northings	= 236 879.56 N

(i) = -0.00206073049599361
(ii) = 3.00397954545871E-06
(iii) = 1.79858170097256E-08
(iv) = -3.2602179751047E-11
A = -13117.5119880953
ϕ' = 53^o 22’ 55”.5606

ν = 6391310.78767229
ρ = 6376077.04245292
η^2 = 0.00238920344248461

(XVII) = 1.22694900282341E-14
(XVIII)= 2.53298453304199E-29

Scale Factor at mid point= 1.00 02 09 50
True distance (i) (using SF at mid point) = 18 642.625 m

Applying Simpson’s rule

Scale Factor from first point (see example 4b)	= 1.000 183 36
Scale Factor from second point (see example 4a)	= 1.000 237 76
Scale Factor from Simpson’s Rule	= 1.000 209 85

True Distance (ii) from Simpson’s Rule SF = 18 642.619 m

6.6 Example 6 : Computation of True Azimuth from Grid Coordinates

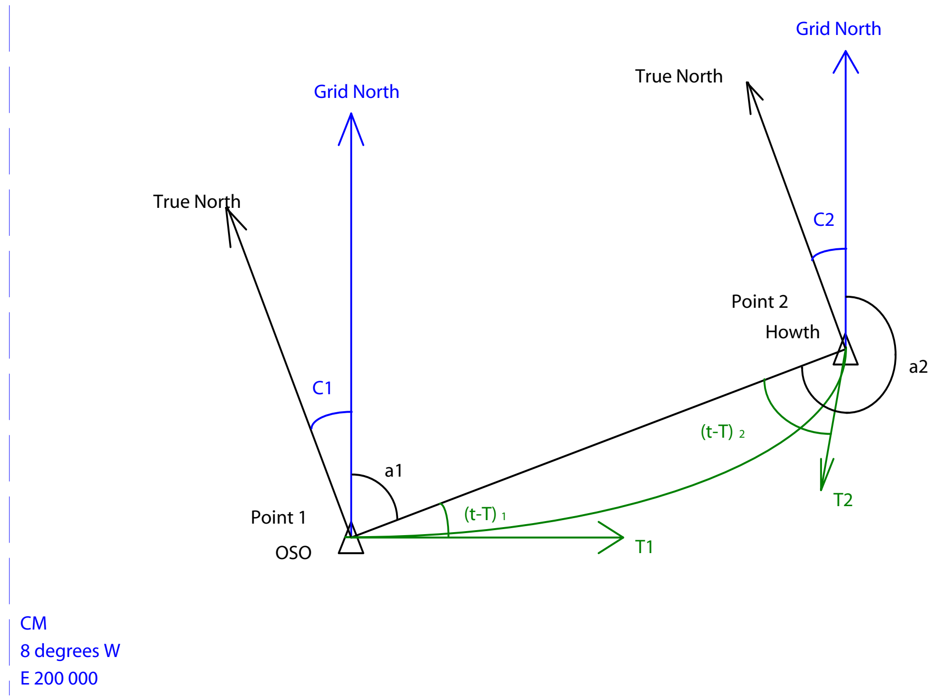
a) Line OSO (Phoenix Park) to Howth

2nd Point	**Howth**	E_2	328 546.34	N_2	237 617.19
1st Point	**OSO**	E_1	309 958.26	N_1	236 141.93
		E_2-E_1	18 588.08	N_2-N_1	1 475.26

Grid Bearing 1 - 2 (a1) = $\arctan\left[\dfrac{(E_2 - E_1)}{(N_2 - N_1)}\right]$ = 85° 27’ 43”.8474

Grid Distance 1 to 2 = $\left[\dfrac{(E_2 - E_1)}{\sin\alpha_1}\right]$ = 18 646.531m

= $\left[\dfrac{(N_2 - N_1)}{\cos\alpha_1}\right]$ = 18 646.531m (check)



The relationship between True Azimuth, Grid Azimuth, Convergence and (t-T) correction for Example 6.

True Azimuth₁ = $a1 + C1 - (t - T)''_1$

True Azimuth₂ = $(a1 + 180) + C2 - (t - T)''_2$

Convergence C at a point

Point 1: OSO (Phoenix Park)

Easting, E = 309 958.26
Northing, N = 236 141.93

(I) = -0.00217658458592961
(ii) = 3.17165352619869E-06
(iii) = 1.89999091950117E-08
(iv) = -3.4425245878838E-11
A = -13854.9696691283
ϕ' = 53^{o} 22' 31".6984

ν = 6391308.41613419
ρ = 6376069.94479899
η^2 = 0.00238994732917419

XXII = 2.10488601181288E-07
XXIII = 4.82209555816333E-21
XXIV = 1.75556049978879E-34

Convergence, C1 at OSO = 01^{o} 19' 32".6690

Point 2: Howth
Eastings, E = 328 546.34
Northings, N = 237 617.19

(I) = -0.00194487610605761
(ii) = 2.83617675824106E-06
(iii) = 1.6972035642991E-08
(iv) = -3.0778062713542E-11
A = -12380.0534863662
ϕ' = 53^{o} 23' 19".4228

ν = 6391313.15904511
ρ = 6376084.13961744
η^2 = 0.00238845960846801

XXII = 2.10590196160653E-07
XXIII = 4.82742602059825E-21
XXIV = 1.75873870923192E-34

Convergence, C2 at Howth = 1^{o} 33' 01".5981
(t-T)

Eastings and Northings of points 1 and 2 as above.
ν = 6391310.78767229
ρ = 6376077.04245292

XXV = 4.08983000941E-15
y_1 = 109958.26
y_2 = 128546.34
N1 and N2 as above

$(t-T)_1$ = - 0".4337
$(t-T)_2$ = + 0".4568

Results:

True Azimuth$_1$		**True Azimuth$_2$**	
a1	85° 27' 43".8474	a2	85° 27' 43".8474
+C1	1° 19' 32".6690	+180°	180° 00' 00".0000
$-(t-T)_1$	-0".4337	+C2	1° 33' 01".5981
		$-(t-T)_2$	+ 0".4568
True Azimuth$_1$	86° 47' 16".9501	True Azimuth$_2$	267° 00' 44".9887

7.0 REFERENCES

[**1**] ORDNANCE SURVEY, 1995. *The Ellipsoid and the Transverse Mercator Projection.* Geodetic Information Paper No 1. OSGB, version 1.2, Southampton

[**2**] BOMFORD G., 1980. *Geodesy*. 4th Edition, Oxford University Press.

[**3**] CLARK, D., revised by JACKSON, J., 1973 *Plane and Geodetic Surveying for Engineers.* Vol 2. 6th edition. Constable & Company Ltd., London.

[**4**] SMITH, J.R., 1997. Introduction to Geodesy. The History and Concepts of Modern Geodesy. John Wiley & Sons, Inc.

[**5**] CLARKE, A.R., 1858. *Account of the Observations and Calculations of the Principal Triangulation; and of the Figure, Dimensions and mean Specific Gravity of the Earth as derived therefrom.* Eyre and Spottiswoode, London.

[**6**] ORDNANCE SURVEY, 1967. *The History of the Retriangulation of Great Britain 1935-1962.* H.M.S.O., London.

[**7**] ANDREWS, J.H., 1975. *A Paper Landscape, The Ordnance Survey in Nineteenth-Century Ireland.* Clarendon Press, Oxford.

[**8**] TAYLOR, W.R. (Lt. Col.), 1967. *An outline of the re-triangulation of Northern Ireland.* H.M.S.O., Belfast.

[**9**] CORY, M.J., 1997. Re-measuring the size of Ireland, Survey Ireland.

[10] ORDNANCE SURVEY OF IRELAND & ORDNANCE SURVEY OF NORTHERN IRELAND, 1999. *Making Maps Compatible With GPS.* Osi Dublin, OSNI Belfast.

APPENDIX A

1.0 OTHER SYSTEMS USED IN IRELAND

NAME	ELLIPSOID	DATUM NAME	DESCRIPTION
Ireland 1965	Airy Modified	Ireland 1965	See Main Report
OSGB70(SN)	Airy	Herstmonceux	The Irish network (1965) was adjusted to OSGB70(SN)[4] junction stations, incorporating additional cross-channel and astronomic observations.
ED50	International Hayford	Helmert Tower, Potsdam	The Irish network (1965) was adjusted to OSGB70(SN) junction stations. ED50 values of Irish triangulation stations obtained by 3D cartesian transformation, derived at Herstmonceux. Unknown stations may be obtained by deriving 3D transformations at nearest three or four surrounding triangulation stations, and applying these to the unknown point.
OS(SN)80	Airy	Herstmonceux	Combined adjustment of Irish and GB terrestrial observations with positions of some stations determined by weighted TRANSIT doppler observations. [5]
ED87	International Hayford	As per ED50	A full readjustment of European terrestrial networks scaled and controlled by space techniques. Observations from Ireland are based on those used in the OS(SN)80 adjustment.
WGS84	WGS84	Earth-centred co-ordinate system	WGS84 is defined by the positions of some 1591 stations observed using TRANSIT doppler observations. Because of this WGS84 is accurate to about 1-2 metres, globally. WGS84 co-ordinates of Irish primary triangulation stations were derived by transformation resulting from TRANSIT doppler observations in the 1980's.

[4] Ashkenazi V., Cross P., Davies M.J.K., Proctor D.W. *The Readjustment of the Retriangulation of Great Britain, and its Relationship to the European Terrestrial and Satellite Networks.* Ordnance Survey Southampton, 1972.

[5] Ashkenazi V., Crane A., Preiss W., Williams J. *The 1980 Readjustment of the Triangulation of the United Kingdom and the Republic of Ireland OS (SN) 80*. Ordnance Survey (Southampton) Professional Paper, New Series No 31.

ETRF / ITRF	GRS80	Earth-centred co-ordinate reference frame.	The European Terrestrial Reference Frame (ETRF) is the realisation of a The European Terrestrial Reference system (ETRS). ETRF is a regional subset of the International Terrestrial Reference Frame (ITRF), which is derived by high precision satellite and space observations at many global geodetic observation facilities. It is thought to be accurate to a few centimetres, which is enough to detect physical movement of stations. Thus annual realisations of positions are possible in this framework, resulting in the necessity of time stamping.

2.0 REFERENCE ELLIPSOIDS

REFERENCE ELLIPSOID	DEFINING PARAMETERS	COMMENTS
International (Hayford)	a = 6 378 388.0m e^2 = 0.006 722 670 022 33	
Airy	a = 6 377 563.3964 b = 6 356 256.9096 e^2 = 0.006 670 540 000 12	
WGS84	a = 6 378 137.000 e^2 = 0.006 694 379 9	There are other defining parameters.[6]
GRS80	a = 6 378 137.000 b = 6 356 752.314 1 e^2 = 0.006 694 380 022 90	Strictly speaking GRS80 is defined by many more constants, however these are not of direct interest in this instance.[7]

[6] Defence Mapping Agency, 1987. *Department of Defense World Geodetic System 1984. Technical Report (and supplements).* DMA TR-8350.2, USA.
[7] Moritz, H., 1988. 'G*eodetic Reference System 1980'*. Bulletin Geodesique, 1988 Volume 62 No 3, Paris.

3.0 PROJECTIONS USED IN IRELAND

Bonne Projection

Projection :	Pseudoconic projection, with true scale on all parallells
Co-ordinates :	Rectangular plane co-ordinates
Units :	Feet of bar O_1
Central Meridian :	08° 00’ 00” W
Standard Parallel :	53° 30’ 00” N
Origin :	Intersection of CM and SP
Radius of mean parallel :	15 516 209.8 feet of Bar O_1
Ellipsoid :	Airy
Limits of System :	Ireland
Properties :	Equal Area
Map scales :	1 inch, 1/2 inch and 9 mile to one inch

Lambert Conformal Conic

Projection :	Conformal conic with two standard parallels
Central meridian :	08° 00’ 00” W
Standard Parallels :	52° 40’ 00” N and 55° 20’ 00” N
Ellipsoid :	International Hayford
Properties :	Conformal
Map scales :	Aeronautical Charts, 1:1 000 000, 1:500 000 and 1:250 000

Cassini Projection

Projection :	Cylindrical projection with line of contact along chosen central meridian
Co-ordinates :	Rectangular spherical co-ordinates
Units :	feet of Bar O1
Central Meridian :	Longitude of Initial points (see **Appendix B**)
Origin :	Different origin for each county (see **Appendix B**)
Ellipsoid :	Oblate Spheroid
Limits of system :	Separate county projections to limit scale errors
Properties :	Not conformal or equal area
Map scales :	County Series 6 inch and 25 inch to one mile

APPENDIX B

ORIGINS OF COUNTY SERIES CASSINI PROJECTIONS.

County	Initial Point	Latitude N	Longitude W	Meridional Distance (m)
Donegal	Letterkenny Church Spire	54° 57’ 02”.84	7° 44’ 18”.53	
Monaghan	Monaghan Church Tower	54° 14’ 53”.06	6° 58’ 07”.38	
Sligo	Cooper’s Observatoryy	54° 10’ 30”.17	8° 27’ 24”.93	19,702,286.200
Mayo	Castlebar Church Tower	53° 51’ 15”.07	9° 17’ 58”.72	
Leitrim	Ck-on- Shannon Church Spire	53° 56’ 48”.43	8° 05’ 37”.74	
Cavan	Cavan Church Spire	53° 59’ 37”.21	7° 21’ 35”.31	
Louth	Dundalk Church Spire	54° 00’ 29”.73	6° 24’ 03”.43	
Meath	Wellington Testimonial	53° 33’ 06”.13	6° 47’ 35”.17	
Westmeath	Mullingar Church Spire	53° 31’ 27”.91	7° 20’ 20”.88	19,464,692.554
Longford	Longford Church Spire	53° 43’ 51”.35	7° 47’ 57”.85	
Roscommon	Roscommon Church Spire	53° 37’ 43”.73	8° 11’ 24”.87	
Dublin	Dublin Observatoryy	53° 23’ 13”.00	6° 20’ 17”.51	19,414,493.354
Offaly	Tullamore Church	53° 16’ 17”.64	7° 28’ 50”.91	
Galway	Galway Church Spire	53° 16’ 20”.49	9° 03’ 11”.60	
Clare	Ennis Church Tower	52° 50’ 44”.07	8° 58’ 51”.44	19,216,822.850
Laois	Port Laoise New Church Spire	53° 02’ 01”.31	7° 18’ 10”.24	
Wicklow	Lugnaquilla	52° 57’ 59”.94	6° 27’ 50”.23	19,261,029.515
Carlow	Mount Leinster	52° 37’ 03”.47	6° 46’ 46”.96	
Kilkenny	Kilkenny Church Spire (St. Mary’s)	52° 39’ 04”.41	7° 15’ 07”.00	
Tipperary	Cashel Church Spire	52° 30’ 53”.48	7° 53’ 06”.71	
Limerick	Rice’s Monument	52° 39’ 27”.61	8° 37’ 40”.72	
Kerry	Tralee Church Spire	52° 16’ 13”.61	9° 42’ 11”.99	19,006,846.490
Cork	Mount Hillary	52° 06’ 34”.72	8° 50’ 20”.50	18,948,141.837
Wexford	Forth Mountain	52° 18’ 56”.00	6° 33’ 42”.37	19,023,314.460
Waterford	Knockanaffrin	52° 17’ 18”.46	7° 34’ 53”.71	19,013,422.930
Kildare	Kildare Round Tower	53° 09’ 27”.68	6° 54’ 41”.86	
Antrim		55° 09’ 42”.69	06° 15’ 00”.71	
Armagh		54° 21’ 10”.53	06° 38’ 55”.82	
Derry		55° 06’ 49”.03	06° 54’ 56”.53	
Down		54° 10’ 48”.28	05° 55’ 12”.05	
Fermanagh		54° 22’ 12”.97	07° 39’ 19”.45	
Tyrone		54° 40’ 26”.38	07° 12’ 31”.47	