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# OSTN02: A NEW DEFINITIVE TRANSFORMATION FROM GPS DERIVED COORDINATES TO NATIONAL GRID COORDINATES IN GREAT BRITAIN

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## ABSTRACT

*The computation of the definitive OSTN02™ transformation is described. A background is given on the horizontal datums in use in Great Britain and on the meaning of a "definitive transformation". The choice of transformation model for OSTN02 is then discussed followed by a detailed account of the data collection, transformation computation and testing. The procedure of building the final transformation data set and delivering it to users is then described.*
*Compared to the previous transformation (OSTN97), OSTN02 contains a substantial increase in stations whose ETRS89 coordinates result from direct occupation with GPS. Testing indicates that OSTN02 has a 97.8% accuracy of 0.2 m. A new transformation data file has been made available to users.*

## INTRODUCTION

Since 1994 Ordnance Survey® has produced an accurate national transformation (OSTN94™) to enable GPS users make their survey data compatible with map data on OSGB36® National Grid [**22**]. This transformation was improved in 1997 (OSTN97™) [**23**]. The introduction of the Ordnance Survey National GPS Network active stations in 2000 has given GPS users increased access to the well-defined GPS compatible coordinate system European Terrestrial Reference System 1989 (ETRS89). Since the ratification of the active network as the official realisation of ETRS89 in Great Britain, the national geodetic coordinate frame has been defined in ETRS89. An improved transformation was therefore required to ensure full compatibility between GPS surveys in ETRS89 and map data on OSGB36 National Grid.

The OSTN02 transformation is based on an extensive ETRS89 survey (described in this paper) of over 3200 existing National Grid control points. OSTN02 also heralds a change in the definition of OSGB36 National Grid. The previous definition was realised by the coordinates of the triangulation stations. Unlike the previous transformations OSTN02 does not just approximate OSGB36 National Grid it now *defines* it, hence it can be considered error free. The OSGB36 National Grid coordinates of an existing OSGB36 point, refixed in ETRS89 and transformed with OSTN02 are now considered "true". The original archived OSGB36 National Grid coordinates of the point will no longer be true, by definition. This change is a subtle change in definition only and will NOT mean that existing OSGB36 coordinates (e.g. of map features) need to be changed in any way for they are still compatible with the new definition of OSGB36 within specified map accuracy tolerances.

## BACKGROUND

### *OSGB36*

OSGB36 is the national horizontal mapping datum of Great Britain and is the basis for the National Grid and all Ordnance Survey map data (except heights which are referenced to Ordnance Datum Newlyn).

GB has had two triangulations observed in the last two centuries. The first - known simply as the *Principal Triangulation* was published in 1858 [**5**]. It was not observed as a single planned scheme but was instead made up in a piecemeal fashion from observations between 1783 and 1853. Two taped bases provided scale while the origin and azimuth were defined at the Royal Observatory, Greenwich. The adjustment was performed by hand in 21 computing blocks using the Airy 1830 ellipsoid.

The second triangulation is known as *The Retriangulation*. It was observed between 1936 and 1953 and computed by hand in 7 blocks. The original origin at Greenwich had been destroyed but was implied by holding the position of 11 stations fixed to the mean of their Principal Triangulation positions. This means that there is no one point that can be described as the origin of the Retriangulation. The coordinates are on the Airy 1830 ellipsoid and this adjustment is known as the "Ordnance Survey of Great Britain 1936 Datum - OSGB36" [**21**]. OSGB36 was realised on the ground by a network of triangulation stations, including the familiar concrete trig pillars on hill tops.

Closely associated with OSGB36 is the National Grid which is realised using a modified Transverse Mercator projection. The term "National Grid" used on its own implicitly refers to the OSGB36 datum. All Ordnance Survey map data coordinates are expressed as eastings and northings in the National Grid [**20**].

OSGB36 has been shown to contain randomly variable scale errors [**3**] and [**2**] and over the entire length of GB the National Grid is approximately 20 m too long. The scale errors are mainly due to OSGB36 being computed in blocks and the fact that scale and azimuth were controlled entirely by the 11 stations from the Principal Triangulation. These scale variations mean that OSGB36 can be described as *inhomogeneous.* The inhomogeneity of OSGB36 does not affect its adequacy as a national mapping datum but it does affect the type of transformation that can be accurately computed.

### *ETRS89*

ETRS89 (European Terrestrial Reference System 1989) is a stable, *homogenous*, geocentric coordinate system that is compatible with the system used by GPS - WGS84 (World Geodetic System 1984). Unlike WGS84 however, ETRS89 is fixed in time and is accurately (≤ 1 cm) realised on the ground. ETRS89 has been adopted as the primary system for coordinate positioning across Europe and the associated ellipsoid is GRS80 [**19**].

ETRS89 in GB is realised by The National GPS Network active stations – a network of 31 (at time of writing) continuously operating GPS receivers (COGRs) – see www.gps.gov.uk. The active network is supplemented by a passive network of approximately 900 ground markers also coordinated in ETRS89. The active network realisation of ETRS89 has been ratified as the official definition of ETRS89 in GB and is known as EUREF GB 2001 [**15**]. The passive network has been recently recomputed with EUREF GB 2001 acting as control and this realisation is known as OSGPS2002 [**16**].

ETRS89 has been adopted as the primary coordinate system in Great Britain. As well as providing a compatible coordinate system for GPS users, this allows an accurate and stable transformation to OSGB36 National Grid to be developed.

*What is a Definitive Transformation?*

Since the Retriangulation, the OSGB36 National Grid coordinate system was realised by the coordinates of the trig points across the country. Since ETRS89 has been adopted as the primary high-order coordinate system in GB, the proposal for OSTN02 was to redefine OSGB36 via ETRS89 plus a transformation. In other words, instead of the transformation just approximating OSGB36, it is defined to be error free and part of the definition of OSGB36. This change means that, for example, the National Grid coordinates of an existing OSGB36 point, refixed using GPS from the National GPS Network plus OSTN02 will be considered to be the correct ones. The original archived OSGB36 National Grid coordinates of the point will no longer be true, by definition.

The important thing to consider in implementing this change is that it is a subtle change in definition only and will NOT mean that existing OSGB36 coordinates of map features need to be changed in any way. I.e. the effect on the user should be minimal in that the difference between OSGB36 coordinates from the transformation and OSGB36 coordinates from the old trig network should be within the accuracy of the mapping. For this reason Ordnance Survey decided to retain the OSGB36 name for the datum.

For OSTN02, it was decided by Ordnance Survey to aim for an overall rms error vector of 0.1 m when comparing OSGB36 coordinates from GPS plus OSTN02 to existing archive OSGB36 coordinates. This accuracy level is well inside the absolute accuracy of the 1:1250 scale mapping, which is approximately ±0.5 m rms [**25**]. The accuracy of the previous transformation – OSTN97 – was 0.2 m rms.

When discussing a national transformation between GPS and the mapping system the question about whether to transform the mapping invariably arises. Many developed countries are in the process of changing their maps to be directly compatible with GPS. This can be considered to be adopting GPS coordinates 'by the front door'. Ordnance Survey decided not to do this. Instead, redefining the National Grid by the OSTN02 transformation of GPS coordinates is adopting GPS coordinates 'by the back door'. As long as the transformation is ubiquitous and standard, the effect is the same.

## CHOICE OF TRANSFORMATION MODEL FOR OSTN02

The choice of model for an accurate ETRS89 to OSGB36 transformation in Great Britain is influenced by the need to model the inhomogeneity of the OSGB36 coordinate system to the required level of accuracy.

A simple Helmert seven parameter type of transformation will not model the inhomogeneity of OSGB36 sufficiently to realise a transformation accurate to 0.1 m [**24**]. Polynomial transformations (sometimes known as 'Multiple Regression') can successfully model inhomogeneity, but it has been shown [**12**] that a polynomial transformation is unsuitable for high accuracy use in GB. The best accuracy that could be achieved was 0.16 m rms with a $5^{th}$ order polynomial. Although this *almost* meets the accuracy requirements there were also problems with highly erratic behaviour near the transformation boundaries and the risk of uncontrolled oscillation between data points.

The grid look-up transformation model has been used before by Ordnance Survey for previous accurate ETRS89 to OSGB36 transformations (models OSTN94 and OSTN97). In order to have the best chance of achieving 0.1 m accuracy and to provide continuity with previous transformations, it was decided by Ordnance Survey that OSTN02 would also be a grid look-up type of transformation model.

The grid look up transformation models the changing differences between two coordinate systems by dividing the transformation into regions and only applying particular parameters to particular regions. The regions are usually cells in a regular grid covering the area in which the transformation is to be applied. For continuity with previous models it was decided that OSTN02 would retain the same 1 km grid spacing used in OSTN94 and OSTN97.

Commonly, the grid look up method uses map grid coordinates and applies the transformation as linear shifts to the eastings and northings of one system to obtain eastings and northings in another system. The method can be divided up into two parts, transformation parameter generation and user access to the transformation using some algorithm.

The network of points used to compute the transformation is divided into Delauney triangles to create a Triangulated Irregular Network (TIN) with the apex of each triangle on a network point. Within each triangle, the transformation can be expressed using an affine linear model. The affine model was chosen because it enables the random scale variations, which contribute to the inhomogeneity of OSGB36, to be modelled along both axes.

The model can be expressed as follows [**26**]:

$$\begin{bmatrix} x_B \\ y_B \end{bmatrix} = \begin{bmatrix} \mu_x \cos\theta_x & \mu_y \sin\theta_y \\ \mu_x \sin\theta_x & \mu_y \cos\theta_y \end{bmatrix} \cdot \begin{bmatrix} x_A \\ y_A \end{bmatrix} + \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \quad (1)$$

where: $x_A, y_A$ = plane coordinates in system A;
$x_B, y_B$ = plane coordinates in system B;
$\mu_x, \mu_y$ = scale factors for x and y axes;
$\theta_x, \theta_y$ = rotations of x and y axes;
$\Delta x, \Delta y$ = shifts along x and y axes.

The six unknowns - ($\mu_x, \mu_y, \theta_x, \theta_y, \Delta x, \Delta y$) can be solved uniquely from the three vertices of each triangle. Hence for any point within the triangle, with coordinates in system A, the coordinates in system B can be calculated.

To apply the transformation as triangles would be very difficult for the user since the task of identifying the triangle that a particular point falls in would require a complicated search algorithm. Further more all the information about the triangles and their associated parameters would have to be stored [**1**]. Another disadvantage of expressing the transformation as triangles is that it is not 'unique', due to the discontinuity in the parameters at the edge of each triangle. Uniqueness is the property where, if a point to be transformed falls exactly on a triangle edge, the resulting interpolated transformation is the same no matter which one of the boundary triangles is used to supply the parameters.

The transformation process is simplified by projecting a regular grid onto the triangles and using the triangle parameters to compute the coordinate shifts at each grid intersection. The expression of the transformation as a grid does "dilute" the accuracy of the transformation slightly but the advantage is that the discontinuities at the edges of the triangles are smoothed out. The transformation is then accessed by interpolating between shifts at grid intersections to obtain shifts at a specific point. Using a bi-linear polynomial to carry out the interpolation ensures that the uniqueness in the transformation is retained [**12**].

The bi-linear interpolation can be expressed as follows [**1**]:

$$s_e = Ae_A + Bn_A + Ce_An_A + D$$

(2)

$$s_n = Ee_A + Fn_A + Ge_An_A + H$$

(3)

where: $s_e$, $s_n$ = coordinate shifts (from system A to B) at a point (eastings and northings shifts respectively);
$e_A$, $n_A$ = coordinates of the point in system A;
$(A, B, C, D, E, F, G, H)$ = polynomial coefficients.

Using the $s_e$ and $s_n$ shifts at the four grid corners - the polynomial coefficients for a grid square can be calculated. The coefficients are then substituted back into equations (2) and (3) to interpolate the shifts at any point within the grid square. Equations (2) and (3) can be expanded to provide single "one step" equations for both shifts [**12**]:

$$s_e = (1-t)(1-u)s_{e0} + t(1-u)s_{e1} + tus_{e2} + (1-t)us_{e3}$$

(4)

$$s_n = (1-t)(1-u)s_{n0} + t(1-u)s_{n1} + tus_{n2} + (1-t)us_{n3}$$

(5)

where: $s_e$, $s_n$ = coordinate shifts (from system A to B) at a point (eastings and northings shifts respectively);
$s_{e0}, s_{e1}, s_{e2}, s_{e3}$ = eastings shifts at corners of grid cell (0 = SW, 1 = SE, 2 = NE, 3 = NW);
$s_{n0}, s_{n1}, s_{n2}, s_{n3}$ = northings shifts at corners of grid cell (0 = SW, 1 = SE, 2 = NE, 3 = NW);
$t = ( e_A - e_{0A}) \div$ width of a grid cell;
$u = ( n_A - n_{0A}) \div$ width of a grid cell;
$e_A$, $n_A$ = eastings and northings, in system A, of point being transformed;
$e_{0A}$, $n_{0A}$ = eastings and northings, in system A, of SW corner of grid cell.

Using either of the above methods, the final transformed plane coordinates in system B are obtained from:

$$e_B = e_A + s_e$$

(6)

$$n_B = n_A + s_n$$

(7)

The advantage of this transformation method is that it will tend to provide the highest accuracy (particularly if the grid is small) but it will reach a point where the accuracy of the transformation matches the characteristic accuracy of the data from which it is calculated. Once this point is reached, further reductions in the grid size will not improve the transformation accuracy. This was demonstrated for the Ireland 75 to ETRF89 transformation [**1**].

The main disadvantage of the method is that it can require the storage of a lot of parameters - the number increasing rapidly as the grid size decreases. Dense grids can only be usefully accessed by computer. Also, the user's access to the transformation, via the interpolation algorithm, is a little more complicated than using a straight-forward set of equations, as in Helmert or polynomial transformations. However this can also be computerised for the user.

## DATA COLLECTION AND PREPARATION

In order for OSTN02 to achieve an accuracy of 0.1 m rms the transformation data set needed to be both improved and expanded to provide a better definition of

ETRS89. For the previous OSTN97 and OSTN94 transformations, only approximately 180 of the 3300+ points in the transformation network had ETRS89 defined by GPS – the rest coming from a readjustment of the Retriangulation observations in ETRS89. For OSTN02 Ordnance Survey decided to run a specific survey campaign to coordinate many more existing OSGB36 points in ETRS89 using GPS.

The data for OSTN02 came from five sources (detailed in the following section):

- a dedicated Definitive Transformation survey team,
- OSTN94 and OSTN97 transformation test points (TTPs) surveyed by field GPS mapping teams (and some from contractors),
- passive network stations collocated with existing OSGB36 trig markers,
- the ETRS89 readjustment of the Retriangulation observations,
- boundary points created around the extreme edge of the grid to ensure coverage of the whole area by the TIN.

By the end of the project the project data store contained 3279 OSGB36 points which also had ETRS89 coordinates directly observed with GPS. This number of directly observed GPS points was a vast improvement on the 180 (approx.) used for OSTN94 and OSTN97. These points were supplemented with a further 1010 OSGB36 points with ETRS89 coordinates from the readjustment of the Retriangulation observations

*Definitive Transformation Survey Team*

To obtain a good spread of points across GB an initial sweep was planned that aimed to survey at least six evenly spaced points in each 1:50000 Landranger map sheet. The OSTN97 transformation test points (TTPs) surveyed so far (see next section) were used to highlight the areas where OSTN97 did not meet its 0.2 m accuracy criteria (so called "hot spots") and direct the survey to concentrate on these particular areas.

A dedicated Definitive Transformation survey team of three surveyors was formed and the GPS observations at OSGB36 points began in October 1999. This team's sole responsibility was the planning, organisation and execution of surveys to coordinate OSGB36 points in ETRS89. To speed production along even further, the team was often supplemented by staff from the Ordnance Survey's field GPS mapping teams.

As the survey started to produce an even distribution of points across the country, a further selection criterion was introduced to help further densify the point set. It was decided that no urban area should be more than 10 km away from a transformation point observed by GPS (i.e. points from the Retriangulation readjustment did not count towards this criterion). The reasoning behind this is that, even in the OSTN97 hot spots, the shift values in the transformation do not change rapidly with distance. A transformation data point every 10 km was seen as a good sample size to model the changing transformation parameters. However, it would have been inefficient to slavishly follow the "10 km rule" across remote rural and mountainous areas where the need for the utmost transformation accuracy is diminished, so the urban areas were introduced into the rule. "Urban" in this regard came from an Ordnance Survey data set that classified any built up area larger than a hamlet as urban.

Specific sub campaigns were carried out to cover some of the more remote areas of the country. Points in the Western Isles, Orkneys and Shetland were observed as part of a Geodetic Surveys & Computations team (GSC) campaign to extend the passive station network into these areas. Some of the remotest islands, i.e. St. Kilda, Sule

Skerry, North Rona and The Flannan Isles were accessed using a helicopter. The Scottish Highlands and Grampian Mountains were also surveyed using a helicopter and due to a fantastic team effort, between GSC and The Definitive Transformation team, 72 extremely remote points were observed in 12 days.

The Foot and Mouth (F&M) disease outbreak in the first half of 2001 threatened to severely restrict the survey work but the team were able to plan around the infected areas, putting some points on hold and bringing other areas forward in the timetable. Some of the last points (and the last Landranger sheet) to be surveyed were on the Isle of Man which was effectively closed for survey work for the entire duration of the F&M outbreak.

The sweep of Landranger sheets was complete by December 2001 and infill survey to comply with the "10 km rule" was completed by the end of March 2002.

*Field GPS Mapping Team OSTN94 and OSTN97 Transformation Test Points*

The OSTN94, and later the OSTN97, transformations were adopted by the field GPS teams for the computation of control points, to be later occupied by total station for the fixing of map detail. However, because neither OSTN94 nor OSTN97 were definitive and because of the known hot spots, any scheme observed by these teams also incorporated at least one transformation test point (TTP). A TTP could be any point suitable for GPS that had also been instrumentally coordinated in OSGB36, e.g. trig pillar or control traverse station. The TTP acted as a check on the OSTN94 or OSTN97 transformation in the area of the survey and if the vector error between the transformed and archived OSGB36 values was greater than 0.2 m (a "fail"), then the OSTN94 / OSTN97 values were rejected and a local Helmert transformation computed in the GPS processing software (by observing some more TTPs). The local transformation would then be used by the field surveyor to supply coordinates of the control points.

All the TTP data - GPS coordinates and the *archive* OSGB36 coordinates of the points, were an important contribution to the OSTN02 transformation data set but the ones used in local transformation computations were even more important. New detail observed in these areas would be directly related to OSGB36 as defined by the TTPs used in the local transformation computation. To ensure that the new transformation correctly modelled the datum differences in these areas it was therefore vital that the coordinate data from these TTPs were in the OSTN02 data set.

Since TTPs were a by-product of the field GPS survey process, their distribution was not planned so they tended to concentrate in the urban areas where most of the field work was conducted.

A further source of TTPs was from contractors observing control points for aerial photography. As part of the contract conditions, any survey of air points using GPS had to follow the same methodology as that employed by the field GPS mapping teams. The TTPs from contractors were collated by Quality Assurance (QA) team as part of their checks on the contract deliverables.

*Passive Network Stations at OSGB36 Trig Markers*

The passive network (OSGPS2002) [**16**] contains 254 points that utilise existing OSGB36 markers. These points therefore have coordinates in both ETRS89 and OSGB36 National Grid and were useful additions to the OSTN02 data set.

*ETRS89 Readjustment of Retriangulation observations*

Previous versions of the transformation – OSTN94 and OSTN97 took the majority of their ETRS89 coordinates from a readjustment of the primary and secondary Retriangulation observations. It is quite likely that most of the weaknesses in these transformations came from short-comings in the readjustment. In order for OSTN02 to achieve an accuracy of 0.1 m, it was obvious that the readjustment alone could not be relied upon to define ETRS89, hence the data collection exercise described above. It was felt however that, despite some drawbacks and issues surrounding the use of these observations, the readjustment could still make a valuable contribution to the ETRS89 definition by adding a significant number of points to the data set. The readjustment was retained because, unlike OSTN94 and OSTN97, OSTN02 had the large directly observed data set of ETRS89 points that could be used to test the readjustment's realisation of an ETRS89 to OSGB36 transformation.

The observation set for the readjustment consists of over 21,400 direction observations and nearly 2000 distances, connecting 3330 primary and secondary triangulation points. The observations had been keyed into electronic form for the scientific readjustments of the primary triangulation [**3**], [**2**] and later the secondary triangulation [**27**].

The main drawback in trying to realise ETRS89 coordinates using the readjustment is that the observations are reduced to the OSGB36 datum on the Airy 1830 ellipsoid. The ETRS89 is associated with the GRS80 ellipsoid, and the difference between the two must be accounted for in order to realise accurate ETRS89 coordinates. The different datum issue for the Retriangulation readjustment has been dealt with extensively in [**12**], where it was shown that the angular corrections – deviation of the vertical, skew normals and normal section / geodesic were insignificant. These corrections were not applied when the observations were first used and need not be applied this time. However the reduction of the distances to the Airy 1830 ellipsoid was a significant change so the distances would need "re-reducing" to GRS80.

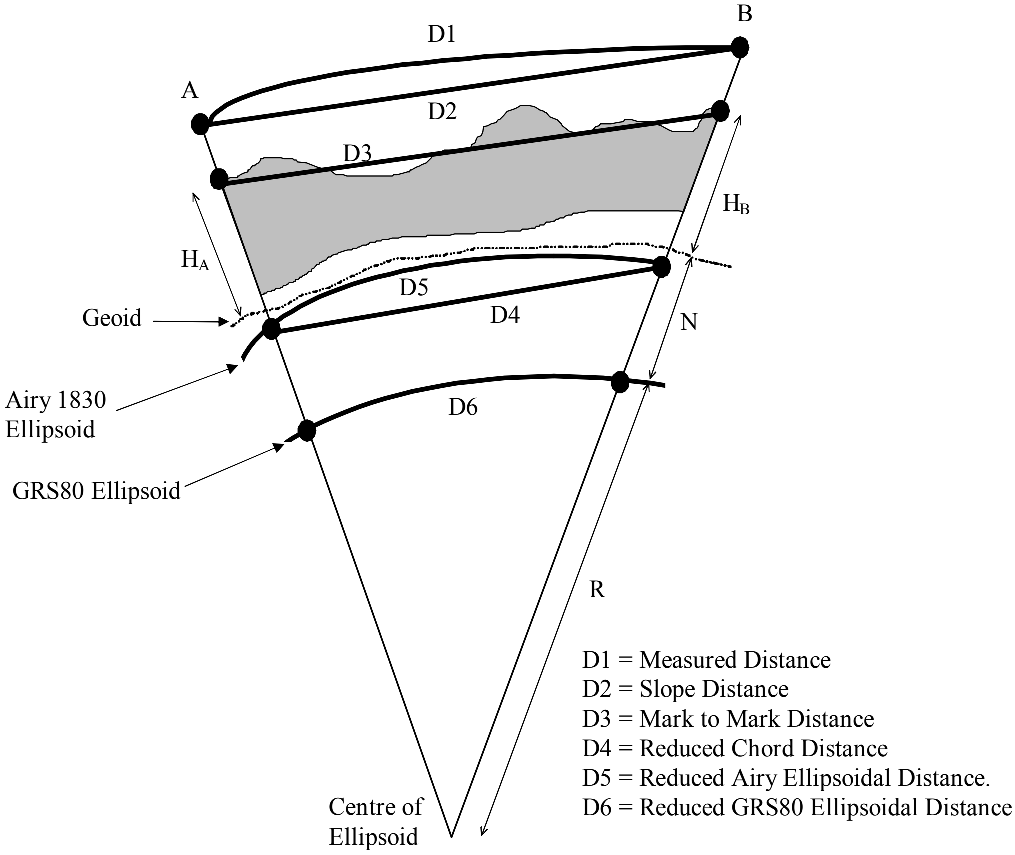


Fig. 1. Reduction of distances to the ellipsoid

Figure 1 shows the relationship of the various distances between two points A and B. The measured distance (D1) is first reduced to slope distance (D2) and then mark to mark distance (D3). It is adequate, for most work on the Airy 1830 ellipsoid, to then reduce to chord distance D4 and arc distance D5 but using the orthometric (above geoid) heights of A and B ($H_A$ and $H_B$ in Figure 1). This is because $H_A$ and $H_B$ are much more likely to be known either from spirit levelling or, for standard mapping work, it is acceptable to obtain $H_A$ and $H_B$ from the contours of a 1:10000 map. Also, the Airy 1830 ellipsoid is positioned such that in GB the separation between it and the geoid is no more than 4.5 m (and in many areas is much less than this) which introduces an error of less than 1 ppm for distances reduced to the geoid but assumed to be on the Airy ellipsoid (D5).

The distances in the data set are on the Airy 1830 ellipsoid (D5) [**2**] and the software used for the readjustment expects distances to be mark to mark (D4). The readjustment is 2D and all points are fixed to zero orthometric height so the geoid / ellipsoid separation (N) from a GRS80 geoid model could then be used to reduce the distance (D4) to the GRS80 ellipsoid (D6). This reduction is actually a close approximation to the true GRS80 ellipsoid distance since it ignores the fact that D4 is mark to mark on the Airy Ellipsoid, instead of being mark to mark on the geoid. However, as stated above, this introduces an error of less than 1 ppm into the reduced distance.

The following formula [**6**] was used to correct arc distances (D5) to chord distances (D4):

$$D4 = 2R\sin\left(\frac{D5}{2R}\right)$$

(8)

where: $R$ = radius of ellipsoid in azimuth of line.

It has been shown [**12**] that it is adequate to use the mean of the ellipsoid axes *a* and *b* as a value for R since over the longest distance observation in the data set (114.84 km), the difference in the correction between using a mean value for R and a rigorous value amounts to only 7 mm. Scale bias parameters were introduced into the distance data sets to account for the known scale errors in some of the early microwave distance measurements [**2**].

For the reduction of the distances D4 to the GRS80 ellipsoid (D6), within the adjustment software, a geoid model was required to give N. At this stage in the project the OSGM02 model [**11**] was not yet complete and the existing OSGM91 model did not cover the entire extent of the observations. Therefore geoid data were obtained from the EGG97 European quasi geoid model [**10**] and a subset covering the entire GB was created using tools in the adjustment software.

A total of 465 points from the OSTN02 data store were also points in the readjustment. These points were fixed in 3D (zero orthometirc height) to define ETRS89 for the readjustment, and 465 was a significant improvement over the 180 used previously for OSTN97 and OSTN94. The 465 fixed points left 2865 free points to be coordinated in ETRS89 by the readjustment.

Not all the free points in the readjustment had final archive OSGB36 coordinates. Some of these points were auxiliary points, long since destroyed roof stations etc. A cross check was done against the full trig archive to extract the free points that did have an OSGB36 coordinate. The check showed that just under 96% (2744 from 2865) of free points had a matching archive coordinate.

Since the observations had already been tested and rigorously filtered for outliers in [**3**], [**2**] and [**27**], it was decided that the best way to analyse the adjustment results would be to test the quality of the ETRS89 to OSGB36 transformation produced from them. This was done by computing a transformation using only the readjustment results and then testing it using all the directly observed GPS points from the OSTN02 data store. The transformation computation and testing methodology is given in the next section. The results of the transformation test are given in Table 1.

Table 1. *Retriangulation adjustment. Difference (m) between transformed and archive OSGB36 coordinates at test points*

| | Eastings | Northings | Vector |
|---|---|---|---|
| Min. | -0.453 | -0.863 | 0.001 |
| Max. | 2.553 | 2.834 | 2.834 |
| Mean | 0.021 | 0.004 | 0.103 |
| RMS | 0.132 | 0.147 | 0.197 |

The areas where the transformation test results indicated the transformation accuracy was less than 0.1 m were analysed using the MapInfo Professional® software and the points from the readjustment within these areas were removed from the final data set. The selection process was over cautious to minimise the risk of poorly defined ETRS89 points from the readjustment affecting the quality of the final transformation. A total of 1734 points were rejected from the data set, leaving 1010 to go into the OSTN02 data store. The large number of points rejected indicates that in some areas the readjustment is incapable of producing ETRS89 coordinates of sufficient quality to meet the 0.1 m accuracy target.

*Boundary Points*

The final points to be added to the OSTN02 data store were some around the outer edge of the extents of the transformation grid. These points were required to force the transformation Triangulated Irregular Network (TIN) to stretch over the whole area of the grid. This would ensure that the transformation grid would be fully populated when sampled from the final TIN.

A point set was created in ETRS89 with the points spaced 50 km outside the grid boundary and with 100 km between each point. E.g. the south west corner boundary point had ETRS89 grid coordinates (produced using National Grid projection on ETRS89) of -50000,-50000 and the point immediately north had coordinates -50000, 50000 and so on. A total of 46 points were created and they are illustrated in Figure 2.

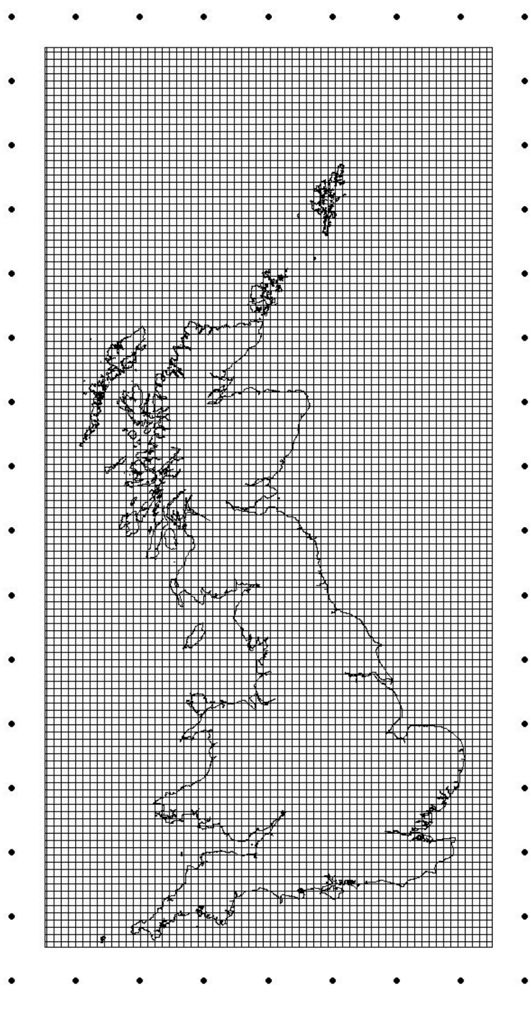

Fig. 2. Boundary points

*Note – the density of the grid shown is diagrammatic only and simply represents the total extent of the transformation grid.*

The OSGB36 coordinates for the boundary points were computed using the Ordnance Survey's published seven parameter Helmert transformation [**24**]. This transformation is only accurate to approximately 5 m, but it is the only one available that could compute OSGB36 coordinates so far from the GB land mass.

## TRANSFORMATION COMPUTATION

The final data set for the OSTN02 transformation computation contained a total of 4289 points spread across the entire country. Figure 3 shows the distribution of the points.

The data set was collated in a spreadsheet and, for each point, the plane ETRS89 coordinates (using National Grid projection) and the shifts *from* ETRS89 *to* OSGB36 were computed. Each point was then assigned a random ID number (between 1 and 4289). The points were then sorted by these ID numbers and the data split into quartiles. The quartiles were used as four new sub data sets each one containing 75% of the total number of points. I.e. data set #1 contained all the points in quartiles 2, 3 and 4; data set #2 contained all the points from quartiles 1, 3 and 4, and so on. This splitting of the data set was for testing purposes, which are described in the next section.

Fig. 3. Final transformation data set.
***Triangles represent points with ETRS89 coordinates directly observed with GPS. Dots represent points from the Retriangulation readjustment.***

Whether computing a transformation from a 75% data set or the final full data set the same methods were used.

- The data was used to create a MapInfo® table and then the points were plotted using their ETRS89 plane coordinates (from National Grid projection).

- Vertical Mapper™ module v2.5 was used in MapInfo Professional v7.0 to create TINs from the points. One TIN was created for the shifts in eastings and another for the shifts in northings. The TINs were created using the affine linear method given in Equation (1).
- A "blank" 700 km x 1400 km grid with a 1 km resolution was created in a text file by listing the ETRS89 plane coordinates of the desired grid nodes.
- The Point Inspection tool of Vertical Mapper was then used to sample the transformation TINs at the coordinates from the text file and export the resulting grid of shifts to another text file.
- The exported text file could then used in software to create a transformation.

## TRANSFORMATION TESTING

### *Accuracy Statement*

The most important aspect of the transformation to be tested is its accuracy. It is wise to define exactly what is meant by accuracy in this case.

**The accuracy of OSTN02 refers to the difference between the coordinates of a point, observed by GPS and transformed to OSGB36 via OSTN02, and the coordinates of the same point observed by terrestrial methods from the local traditional control. This meaning of accuracy is further qualified by quoting the lowest order of traditional control to which it applies. In the case of OSTN02 the accuracy statement applies to traditional control points down to fourth order since this is the lowest order of control against which OSTN02 has been tested.**

The accuracy of OSTN02 was tested in two ways - using internal data and using an external set of test points observed by Quality Assurance team.

### *Internal Testing*

A transformation can be tested by applying it to other points that have coordinates in both systems, but are not part of the transformation data set. The resulting differences between the transformed and observed coordinates indicate the accuracy of the transformation. However, if other points with coordinates in both systems are available then ideally, they should be in the transformation data set to make it as accurate as possible. An accepted compromise is to initially reserve a random portion (usually 25%) of the final data set for testing purposes, i.e. these points are not used in the transformation computation. When the testing is complete the reserved points are put into the data set and the final transformation is computed.

For OSTN02 a process where all the data could be tested was employed. Instead of creating one random 25% portion of the data set four random quartiles were created. Each quartile could then be tested against a transformation computed from the other three quartiles. Doing this four times rather than just a single 75% / 25% random split means that every point was tested and a clearer picture of the transformation accuracy should emerge.

The results of the tests on the four data sets were plotted in MapInfo Professional. The magnitude and direction of the transformation vector errors for each point were plotted and the magnitudes of all the vectors were also represented by a coloured surface. This made it easy to see areas of large error that required further investigation.

Any point in a "warm" area (i.e. error approaching or greater than 0.2 m) of a test plot was closely investigated in relation to its surrounding test points and the points from which the relevant transformation parameters had been computed. The points order, history and survey source were also taken into account in deciding if it was an outlier or just represented an area of high fluctuation in the transformation shifts and should therefore stay in the data set (to model the fluctuations as closely as possible).

Some points on the edge of the data set displayed large errors. In these cases, the error was due to the fact that the point being tested was the only one in that area and the transformation was being defined by other points far away and by boundary points (generated from the less accurate seven parameter transformation). In these cases the cross section analysis tool in Vertical Mapper was used to study the change in the transformation shifts out towards the boundary points. The effect of poorly estimated shifts at boundary points, on the transformation shifts within the outline of GB, was then minimised by altering the boundary point shifts to produce a smoother cross sectional profile.

A total of 16 points were deleted from the data set after the first analysis of the test plots. The amended data sets were then renamed #1a, #2a, etc and the testing process repeated. A further 4 points were deleted as outliers following this second round of testing.

The tests results for all four data sets after outlier points had been removed are given in Table 2. The test results show that, over all the testing data sets, the vector RMS error is just over 0.08 m which is inside the target value for OSTN02 of 0.1 m. Table 3 shows the percentage of points with a vector error of a certain magnitude.

Table 2. *Internal testing. Difference (m) between transformed and archive OSGB36 coordinates at test points.*

| | Data Set #1 | | | Data Set #2 | | |
|---|---|---|---|---|---|---|
| | Eastings | Northings | Vector | Eastings | Northings | Vector |
| Min. | -0.426 | -0.271 | 0.002 | -0.216 | -0.296 | 0.003 |
| Max. | 0.246 | 0.232 | 0.430 | 0.424 | 0.338 | 0.446 |
| Mean | -0.002 | 0.001 | 0.065 | 0.000 | -0.003 | 0.066 |
| RMS | 0.060 | 0.056 | 0.083 | 0.059 | 0.058 | 0.083 |
| | Data Set #3 | | | Data Set #4 | | |
| | Eastings | Northings | Vector | Eastings | Northings | Vector |
| Min. | -0.377 | -0.328 | 0.001 | -0.420 | -0.385 | 0.001 |
| Max. | 0.403 | 0.287 | 0.443 | 0.211 | 0.371 | 0.444 |
| Mean | 0.000 | 0.005 | 0.066 | 0.001 | 0.004 | 0.067 |
| RMS | 0.062 | 0.056 | 0.084 | 0.061 | 0.058 | 0.084 |

Table 3. *Internal testing. Percentage of points (across all four data sets) with vector errors of, or better than, a given magnitude*

| Magnitude of vector error (m) | % |
|---|---|
| 0.01 | 5.0% |
| 0.05 | 45.9% |
| 0.10 | 81.1% |
| 0.15 | 93.5% |
| 0.20 | 97.8% |

*External Testing*

In addition to the internal testing, an external test was carried out with points observed by Quality Assurance (QA) team, supplemented with TTPs observed after the transformation computations had begun but before OSTN02 was released. These points were not used in the transformation data set and were not added to the data set after testing.

A total of 145 points were available for the external tests, 83 from QA and 62 from TTPs. The results of the tests are in Table 4. The results of the external test indicate that the magnitude of the rms vector error for OSTN02 is still within the 0.1 m target.

The results are slightly worse due to the test being external but they give a better indication of what can be achieved in practice.

Table 4. *External testing. Difference (m) between transformed and archive OSGB36 coordinates at test points*

| | Eastings | Northings | Vector |
|---|---|---|---|
| Min. | -0.207 | -0.213 | 0.006 |
| Max. | 0.181 | 0.215 | 0.238 |
| Mean | -0.011 | -0.008 | 0.083 |
| RMS | 0.075 | 0.064 | 0.099 |

*Independent Methodology Assessment*

The entire transformation computation methodology was assessed by an external expert. Professor Paul Cross from the Geomatic Engineering Department, University College London carried out a two day audit of the transformation in April 2002. Professor Cross' overall conclusion was that -

*"The proposed methodology is appropriate to the task of defining a transformation that has an RMS accuracy of 10cm"* [**8**].

## FINAL DATA SET CREATION

Once the transformation was complete, the final data set of parameters had to be created for delivery to the users. The previous transformations OSTN94 and OSTN97 only contained horizontal shift parameters, but for OSTN02 it was decided to incorporate the new OSGM02 geoid model [**11**] into the same data set. This was to benefit developers and users by having the data necessary for a 3D transformation in one data set.

The OSGM02 model [**11**] was supplied to Ordnance Survey as a set of geoid data files along with some interpolation software. The whole of the UK and Ireland is covered by a gravimetric geoid prior to referencing to any of the existing height datums in the area. The whole of GB is covered with one file referenced to ODN. There are also 5 "patch" files whose extents fall within the area covered by the main GB file. Four of the patches cover island areas where the geoid heights are referenced to local height datums other than ODN - Isle of Man, Shetland Isles, St Kilda and Outer Hebrides. A fifth patch covers the Orkney Isles, which although referenced to ODN via a trig heighting link across from the mainland, was computed as a separate area to preserve the quality of the main ODN file.

There are also five further island areas in GB that do not have a defined local datum point and no patch within OSGM02 but the ETRS89 heights of any OSTN02 points plus OSGM02 effectively define a new height datum in these areas. The geoid heights in these islands would come from the main ODN patch but be flagged as referring to the local datum. The islands are Fair Isle, Foula, Flannan Isles, North Rona and Sule Skerry.

Finally, the Scilly Isles were a unique case. Only one point on the islands, with an existing local datum height, was occupied for OSTN02 and given an ETRS89 height. This point was used to compute the offset of -0.71 m between the final gravimetric geoid and Scilly Isles but a separate geoid model patch was not created. The Scilly Isles heights were extracted from the gravimetric geoid and reduced by 0.71 m to bring them onto the Scilly Isles datum, before being added to the final data set.

The geoid separations in the final combined data file were flagged according to which height datum they referred to.

For OSTN97, access to the transformation offshore was restricted by a simple boundary polygon programmed into the transformation software, however the raw data set grid for OSTN97 was still fully populated. For OSTN02 it was decided a 10 km offshore transformation boundary would be built into the data set rather than applied in software. The 10 km size of the boundary was arrived at after discussions with the UK Hydrographic Office about the requirements of users in the near shore zone. The OSGM02 parameters would also be confined by the same boundary as OSTN02.

A 10 km buffer was created around GB in MapInfo Professional and overlaid onto the transformation grid. The SQL query facilities in MapInfo Professional were then used to carry out a "point in polygon" test on each grid node. At any node outside the buffer the shift values and the geoid datum flag were set to zero. This effectively restricts the transformation to within the 10 km buffer. It was also realised at this stage that the northern most 150 km of the 700 x 1400 km transformation grid were redundant, consisting entirely of zeroed points. It was therefore decided to trim the final transformation grid down to 700 x 1250 km.

## PROMOTION AND DELIVERY TO USERS

Prior to the completion of the transformation, every opportunity was taken to inform users of the upcoming release and also of the significance of the change to a *Definitive* Transformation. Papers were published in Geomatics World [**13**] (aimed at surveyors), Geo:connexion [**14**] (for the GIS community) and Civil Engineering Surveyor [**17**]. Talks and presentations were also given to The Survey Association, World of Surveying 2002 and at many other user seminars.

Once the final data set was created, it was passed to Quest Geodetic Software Solutions (www.geodetic-solutions.com) for the production of a new version of the Grid InQuest transformation software and dynamic link library (dll). Both the software and dll are available free of charge from the Ordnance Survey National GPS Network web site (www.gps.gov.uk). The dll was also used to upgrade the online transformation facilities on the web site. The raw data file is also available on the GPS web site along with a comprehensive user guide and other related information such as a test data file.

An Ordnance Survey press releases were issued and every opportunity was taken to advertise the transformation internally within Ordnance Survey. The release was announced to the academic community via the JISCmail email lists (www.jiscmail.ac.uk); Geodesy, Geophysics, GIS, Satellite Navigation, Surveying-Teaching. Email announcements and follow up conversations were had with the leading GPS manufacturers and GIS software companies.

The efforts to publicise OSTN02 / OSGM02 and the implications of *Definitive* are ongoing with every opportunity being taken to inform users about the changes to the transformation.

## SUMMARY AND RECOMMENDATIONS

The grid look-up type of transformation is the one most suited to an accurate (0.1 rms) ETRS89 to OSGB36 National Grid transformation [**12**].

The Definitive OSTN02, ETRS89 to OSGB36 National Grid, transformation has been computed and the target accuracy of 0.1 m rms has been achieved.

The methods by which the transformation was computed have been approved by an independent expert.

A combined OSTN02 / OSGM02 data file with an inbuilt 10 km offshore boundary has been produced and incorporated into software and the Ordnance Survey National GPS Network web site.

The introduction of OSTN02 and its role in the definition of OSGB36 is the most important change to Great Britain's mapping datum since its inception. The program of user awareness about this important change must continue to ensure the widest possible take up and understanding of the transformation and its implications.

It should be stressed that ETRS89 coordinates only need to be transformed to OSGB36 National Grid when they have to be displayed against other map data that is also in OSGB36. If there is no need to display the ETRS89 coordinates in this way then they can be left in the ETRS89 coordinate system. For the same reason, all GPS processing and adjustment should take place in the ETRS89. However, the transformation of coordinates from ETRS89 to OSGB36 (if required) should always be the final stage of any GPS survey.

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